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TOPMODEL's Subsurface Flow Sensitivity to Spatially Distributed Groundwater Recharge Rate and Soil Depth

Abhängigkeit der Grundwasserströmung von der räumlich verteilten Grundwasserneubildungsrate und Bodenmächtigkeit in TOPMODEL

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Inhalt

1. Introduction

The physically based distributed models of the water cycle such as CATFLOW (T. MAURER, 1997), IHDM (A. CALVER, 1988) or SHE (M. B. ABOTT et al., 1986) are very complex and need large amount of distributed information. It is recognised that with the present quality of available field data, many parameters will loose some of their direct physical interpretation (K. J. BEVEN, 1989, B. G. GRAYSON et al., 1992). Their utilisation is limited to few really well instrumented experimental watersheds. In most catchments it is thus necessary to develop different hypotheses on the one hand to simplify process description using for example HAUDE formula (W. HAUDE, 1955) for ETP calculation instead of the PENMAN formula (H. L. PENMANN, 1950) and on the second hand to describe the spatial variations of data and parameters like pedo-transfer-functions to obtain spatial soil retention and conductivity curves. Thus this modelling way is characterised by a really complex water cycle process description which is simplified each time when data and parameters are not available.

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Another modelling way is to voluntary simplify the model, i.e. the concept, and increase step by step the perception level by adding information into the model. This is illustrated in this paper by using the conceptual model TOPMODEL (K. J. BEVEN & M. J. KIRKBY, 1979, K. J. BEVEN et al., 1995). This model is semi-distributed: the only spatial information is the topography but all other parameters are lumped. This is of course an important simplification but justified from the TOPMODEL authors view due to the fact that spatial information are often missing. The purpose of this paper is to study the effects of spatially distributed soil depth and groundwater recharge rate information on subsurface flow algorithms and simulations.

2. TOPMODEL presentation

TOPMODEL simulations are based on the variable contributing area concept introduced by P. CAPPUS (1960) and J. D. HEWLETT (1961). The basic assumptions that govern TOPMODEL are:

1. that the dynamics of the saturated zone can be approximated by successive steady state representations,
2. that the hydraulic gradient of the saturated zone can be approximated by the local surface slope, $\tan\beta$,
3. that the downslope transmissivity with depth is an exponential function of storage deficit. Note that this assumption has been relaxed in B. AMBROISE et al. (1996a) with other downslope transmissivity.

The subsurface outflow from each grid cell is then calculated as (for complete development see K. J. BEVEN, 1995):

$$q_i = R_i \cdot a_i = To_i \tan\beta_i \exp\left(-\frac{S_i}{m_i}\right) \tag{1}$$

with:

R_i the local groundwater recharge rate value,
To_i the local transmissivity when soil is just saturated. For a constant saturation conductivity, Ko_i, with soil depth we have $To_i = Ko_i \cdot m_i$,
m_i the local exponential rate of decrease of the saturated conductivity with deficit,
S_i the local saturation deficit,
a_i the upslope area draining through the cell i, and
$\tan\beta_i$ the local slope.

From this double equality we obtain the saturation deficit equation:

$$S_i = -m_i \cdot \ln\left(\frac{R_i \cdot a_i}{To_i \tan\beta_i}\right), \tag{2}$$

$$\overline{S} = -\frac{1}{i_A}\sum_{i=1}^{i_A} m_i \cdot \ln\left(\frac{R_i \cdot a_i}{To_i \tan\beta_i}\right). \tag{3}$$

Furthermore we define the local topographic index λ_i first proposed by M. J. KIRKBY (1975) and the local soil topographic index γ_i proposed by K. J. BEVEN (1986) as:

$$\lambda_i = \ln\left(\frac{a_i}{\tan\beta_i}\right), \quad (4)$$

$$\gamma_i = \lambda_i - \ln To_i \quad (5)$$

and for the mean values:

$$\overline{\lambda} = \frac{1}{i_A}\sum_{i=1}^{i_A}\ln\left(\frac{a_i}{\tan\beta_i}\right), \quad (6)$$

$$\overline{\gamma} = \overline{\lambda} - \frac{1}{i_A}\sum_{i=1}^{i_A}\ln To_i. \quad (7)$$

Using these equations we obtain:

$$\frac{S_i}{m_i} + \gamma_i = -\ln R_i, \quad (8)$$

$$\overline{\xi} + \overline{\gamma} = -\frac{1}{i_A}\sum_{i=1}^{i_A}\ln R_i, \quad (9)$$

with

$$\overline{\xi} = \frac{1}{i_A}\sum_{i=1}^{i_A}\frac{S_i}{m_i}. \quad (10)$$

Finally the subsurface flow expression is:

$$Q_b = \sum_{i=1}^{i_A} l_i \cdot q_i = \sum_{i=1}^{i_A} l_i \cdot a_i \cdot \exp\left(-\gamma_i - \frac{S_i}{m_i}\right). \quad (11)$$

In the original TOPMODEL formulation, because of the lack of spatial data, every parameter is considered spatially constant. This means that $R_i = \overline{R}$, $m_i = \overline{m}$ and $Ko_i = \overline{Ko}$, thus $To_i = \overline{To}$.

For a spatially homogeneous groundwater recharge we have (see the difference eq. 8):

$$\frac{S_i}{\overline{m}} + \gamma_i = \frac{\overline{S}}{\overline{m}} + \overline{\gamma}. \quad (12)$$

Thus, subsurface flow calculated when the parameters are spatially homogeneous is given as:

$$Q_b = A \cdot \exp(-\overline{\gamma}) \cdot \exp\left(-\frac{\overline{S}}{\overline{m}}\right), \quad (13)$$

where A is the watershed area

$$A = \sum_{i=1}^{i_A} l_i \cdot a_i. \quad (14)$$

This concept is really efficient in many cases. The relative simplicity of the TOPMODEL structure certainly explains a part of its success. The topography introduced in the TOPMODEL through the topographic index explains nearly the entire spatial dynamic of the saturated zone. It is clear that topography plays a key role in the water movement in a catchment. But it is not the only controlling factor. This could be the reason why TOPMODEL conductivity is grid scale sensitive (P. QUINN et al., 1991, D. M. WOLOCK & C. V. PRICE, 1994, W. H. ZHANG & D. R. MONTGOMERY, 1994, G. M. SAULNIER, 1996).

TOPMODEL structure only explains a part of the saturated zone variations in space but conceptually it is difficult to separate the temporal watershed dynamic from its spatial component. To improve TOPMODEL spatial representation we will now study first the influence of spatially distributed groundwater recharge rate data and second the influence of spatially distributed soil depth data on subsurface flow simulations.

3. Introducing spatial groundwater recharge data

When local groundwater recharge data are available subsurface flow is calculated as:

$$Q_b = \sum_{i=1}^{i_A} l_i \cdot a_i \cdot \exp(-\gamma_i) \exp\left(-\frac{S_i}{m_i}\right) = \sum_{i=1}^{i_A} l_i \cdot a_i \cdot R_i. \quad (15)$$

To follow TOPMODEL conceptualisation we will introduce a mean value from the local recharge rate denoted $\overline{R}'$. It is important to recognise that this change from local values to a mean value will not affect the two different topographic indexes because both quantities are not calculated using the groundwater recharge rate.

From eq. 9, we see that a possible value for $\overline{R}'$ is the logarithmic mean:

$$\ln\overline{R}' = \frac{1}{i_A}\sum_{i=1}^{i_A} \ln R_i. \quad (16)$$

We note with a prime all the new quantities calculated with the mean recharge rate and we study differences resulting from this modification. Recalling eq. 2 we now obtain:

$$\frac{S'_i}{\overline{m}} = -\ln\left(\frac{\overline{R}' \cdot a_i}{To_i \tan\beta_i}\right) = -\ln\overline{R}' + \ln R_i + \frac{S_i}{\overline{m}}. \quad (17)$$

Thus

$$\frac{S'_i}{\overline{m}} - \frac{S_i}{\overline{m}} = -\ln\overline{R}' + \ln R_i \quad (18a)$$

and the mean deficit storage value is given as:

$$\frac{\overline{S}'}{\overline{m}} - \frac{\overline{S}}{\overline{m}} = \ln\overline{R}' + \frac{1}{i_A}\sum_{i=1}^{i_A} \ln R_i = 0 \quad (18b)$$

choosing $\overline{R}'$ calculated from eq. 16. We recognise that this choice leads to modification of local storage deficit values but not its mean value. Because R_i has been replaced by $\overline{R}'$, the new expression for subsurface flow calculation is:

$$Q'_b = A \exp(-\overline{\gamma}') \exp\left(-\frac{\overline{S}'}{\overline{m}}\right). \quad (19)$$

The groundwater recharge rate values are not used for the soil topographic index calculation and consequently $\overline{\gamma}' = \overline{\gamma}$. The new subsurface flow is calculated as from the original TOPMODEL employing mean values, assuming a homogeneous recharge rate $\overline{R}$. For the subsurface flow difference we have:

$$Q'_b - Q_b = \sum_{i=1}^{iA} l_i a_i \overline{R}' - \sum_{i=1}^{iA} l_i a_i R_i = A\overline{R}' - \sum_{i=1}^{iA} l_i a_i R_i \qquad (20a)$$

with Q_b calculated with the local values and Q_b' with the mean value. Finally we have:

$$Q'_b - Q_b = A\left(\overline{R}' - \overline{R}''\right) \qquad (20b)$$

with

$$\overline{R}'' = \frac{1}{A} \sum_{i=1}^{iA} l_i a_i R_i, \qquad (21)$$

the geographic mean R_i value. Thus a difference will allways exist if we use the logarithmic mean value of recharge rate.
From eq. 20a, we recognise that another mean value for the recharge rate would be $\overline{R}''$. For this new choice we have now (we denote with double prime the new values):

$$\frac{S''_i}{\overline{m}} - \frac{S_i}{\overline{m}} = -\ln\overline{R}'' + \ln R_i \qquad (22)$$

and

$$\frac{\overline{S}''}{\overline{m}} - \frac{\overline{S}}{\overline{m}} = -\ln\overline{R}'' + \frac{1}{iA}\sum_{i=1}^{iA} \ln R_i = -\ln\overline{R}'' + \overline{R}'. \qquad (23)$$

The right expression from eq. 23 is not equal to zero anymore compared to the result from eq. 18b. The choice of $\overline{R}''$ has been made so that the subsurface flow is not modified (see eq. 20a). The subsurface flow is obtained:

$$Q_b = Q''_b = A\ \exp\left(-\overline{\gamma}''\right)\ \exp\left(-\frac{\overline{S}''}{\overline{m}}\right) = A\ \exp\left(-\overline{\gamma}\right)\ \exp\left(-\frac{\overline{S}}{\overline{m}}\right)\ \exp(\ln\overline{R}'' - \ln\overline{R}') \qquad (24)$$

thus

$$Q_b = Q''_b = \frac{\overline{R}''}{\overline{R}'} A\ \exp\left(-\overline{\gamma}\right)\ \exp\left(-\frac{\overline{S}}{\overline{m}}\right) \qquad (25)$$

with $\overline{R}'$ defined in eq. 16 and $\overline{R}''$ in eq. 21.

This new choice of the mean recharge rate value permits to calculate the subsurface flow in a simple manner without major modifications in the TOPMODEL algorithms. Even when the spatial values from the recharge rate are used, it is possible to calculate subsurface flow with $\overline{\gamma}$ and $\overline{S}$ values. Introducing spatial groundwater recharge rate is an important additional information for runoff simulation.

In the catchment Höhenhansl (eastern Styria, Austria) measured soil water content data under forest and meadow shows great temporal differences that could partly explain the spatial variation in the saturated zone. It would now be possible to take the influence of landuse changes on the groundwater recharge into account in the TOPMODEL simulations.

This also means that in the original TOPMODEL formulation, i.e. with homogeneous groundwater recharge, we don't take into account in the $\overline{R''}/\overline{R'}$ ratio or that this ratio is always equal to 1 (compare eq. 13 and eq. 25). In the next part we will consider a spatially homogeneous recharge rate when information about the spatial variation is not available or local values have been replaced by a mean value following procedures discussed before.

4. Introducing spatial soil depth data

The parameter $\overline{m}$ used in TOPMODEL is ambivalent. On the one hand, $\overline{m}$ can be seen as a parameter scaling the recession curve decrease (B. AMBROISE et al., 1996). This means that when the master recession curve for a catchment under study is known, it is possible to set the value of $\overline{m}$ so that there is no need to calibrate it. If only a single event is simulated, the $\overline{m}$ value is determined from the recession curve of that particular event. On the other hand, $\overline{m}$ is the parameter that scales the exponential rate of decrease of the saturated conductivity with increasing deficit. 63 % or the total transmissivity of the profile is above a depth (expressed as a defizit) equal to $\overline{m}$; 86 % of the total transmissivity is above a deficit equal to 2 $\overline{m}$. This justifies that the parameter $\overline{m}$ can be seen as an index of the effective depth for rapid downslope flows (G. M. SAULNIER et al., 1997). This means that also the parameter $\overline{m}$ can vary greatly within the catchment. Let us now illustrate how spatial m_i values would influence the subsurface flow calculation and the TOPMODEL algorithms (again new values are noted with a prime).

Recalling eq. 9 and eq. 13, subsurface flow is now calculated as:

$$Q_b = A\ \exp(-\overline{\gamma})\ \exp(-\overline{\xi}). \tag{26}$$

Remembering the ξ definition, the suitable expression for the m value is:

$$\overline{m}' = \frac{1}{i_A}\sum_{i=1}^{i_A} m_i. \tag{27}$$

The transmissivity obtained by introducing $\overline{m}'$ is:

$$\overline{To}' = To'_i = \overline{Ko}\cdot\overline{m}' = \frac{\overline{m}'}{m_i}\cdot\overline{Ko}\cdot m_i = \frac{\overline{m}'}{m_i}\cdot To_i. \tag{28}$$

Further wie obtain:

$$\frac{S'_i}{\overline{m}'} = -\ln\left(\frac{\overline{R}\cdot a_i}{\overline{To}'\tan\beta_i}\right) = -\ln\left(\frac{m_i}{\overline{m}'}\right) + \frac{S_i}{m_i}, \tag{29}$$

$$\gamma'_i = \ln\left(\frac{m_i}{\overline{m}'}\right) + \gamma_i \tag{30}$$

and for the mean values:

$$\frac{\overline{S}'}{\overline{m}'} = -\frac{1}{i_A}\sum_{i=1}^{i_A}\ln\left(\frac{m_i}{\overline{m}'}\right) + \overline{\xi}, \tag{31}$$

$$\overline{\gamma}' = \frac{1}{i_A}\sum_{i=1}^{i_A}\ln\left(\frac{m_i}{\overline{m}'}\right) + \overline{\gamma}. \tag{32}$$

Thus the subsurface flow is:

$$Q'_b = A \exp(-\overline{\gamma}') \exp\left(-\frac{\overline{S}'_i}{\overline{m}'}\right) = A \exp(-\overline{\gamma}) \exp(-\xi) = Q_b . \tag{33}$$

It can be recognised from eq. 33 that the subsurface value is not modified. This means that we can calculate the subsurface flow using the mean value $\overline{m}'$ and we don't need to calculate the subsurface outflow from each cell.

From eq. 31 and 32 another expression for average $\overline{m}''$ value would be (by comparison with the groundwater recharge rate):

$$\ln \overline{m}'' = \frac{1}{i_A} \sum_{i=1}^{i_A} \ln(m_i) . \tag{34}$$

Local values are still defined with eq. (28), eq. (29) and eq. (30) but for the mean values noted with a double prime there is no difference anymore so that:

$$\frac{\overline{S}''}{\overline{m}''} = \overline{\xi} \text{ and } \overline{\gamma}'' = \overline{\gamma} . \tag{35}$$

Again the total subsurface flow is not modified. The mean value in eq. 34 is estimated better as the one in eq. 27 because more values are unaltered $\overline{\gamma}$, $\overline{S}$ and Q_b. Thus, the algorithms used in the original TOPMODEL version to calculate $\overline{\gamma}$, $\overline{S}$ don't need to be modified so that spatial soil depth could easily be incorporated in TOPMODEL framework.

5. Conclusion

We have illustrated in the present work how spatial parameters could be introduced in the semi-distributed model TOPMODEL. It was the main goal to see their influence on subsurface flow simulation and to keep TOPMODEL algorithms as simple as possible. Two really different results are obtained:

Introducing spatial soil depth has no impact on subsurface flow simulation. The soil depth is needed for the soil topographic index and the saturation deficit calculation. However we have shown that using the mean soil depth value defined in eq. 34 will not modify these two quantities and permits to calculate subsurface flow in a simple manner, i.e. without calculating each grid cell subsurface outflow.

Introducing groundwater spatial recharge modifies subsurface flow calculation in comparison with the original TOPMODEL formulation, i.e. with homogeneous groundwater recharge rate. For this reason, we should use spatial information that could be gained for example through landuse cartography.

Thus TOPMODEL could be of more widespread applicability if the groundwater spatial pattern would be taken into account. The subsurface flow calculation algorithm would still be simple enough because the only modification is the introduction of the ratio: $\overline{R}''/\overline{R}'$ (see eq. 25). This ratio also illustrates the error that is made at each time step in the original TOPMODEL for subsurface flow calculation with spatially homogeneous groundwater recharge values.

Detailed evaluation of this error will be done for the Höhenhansl catchment. It is expected that this error will be of less importance for the winter months, i.e. when the vegatation has little water demand. For the summer months, on the contrary, we believe that the spatial groundwater recharge will strongly influence the spatial groundwater dynamic and the expected error will then be important.

Summary

The semi-distributed rainfall-runoff model TOPMODEL only uses the topography as spatial information. All the other data and parameters are lumped over the catchment. We have developed a theoretical method to introduce spatial groundwater recharge rate and effective soil depth to extent TOPMODEL applicability. The subsurface flow sensitivity to this changes has been observed. Furthermore, the new algorithms have been kept as simple as possible so that these spatial information could easily be integrated in the TOPMODEL structure for subsurface flow calculation.

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Zusammenfassung

Das „semi-distributed rainfall-runoff TOPMODEL" benötigt nur die Topographie als räumliche Information. Alle anderen Daten und Parameter werden als Mittelwert des Einzugsgebietes herangezogen. Wir haben eine theoretische Methode entwickelt, indem wir die Grundwasserneubildung und die effektive Bodentiefe eingeführt haben, um die Verwendung von TOPMODEL zu erweitern. Die Sensibilität des modellierten unterirdischen Abflusses zu diesen Veränderungen wurde beobachtet. Die neuen Algorithmen blieben so einfach wie möglich, sodaß man die räumliche Information ohne größeren Aufwand in das TOPMODEL integrieren kann, um den unterirdischen Fluß zu berechnen.

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