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CONSTITUTION OF THE INTERIOR OF THE EARTH.

[Quart. Journ. Geol. Soc., vol. lxii, 1906,
pp. 456–473.]

[*From the* QUARTERLY JOURNAL *of the* GEOLOGICAL SOCIETY *for August* 1906, Vol. lxii.]

CONSTITUTION OF THE INTERIOR OF THE EARTH.

The CONSTITUTION *of the* INTERIOR *of the* EARTH, *as* REVEALED *by* EARTHQUAKES. By RICHARD DIXON OLDHAM, F.G.S.

I. INTRODUCTORY.

OF all regions of the earth none invites speculation more than that which lies beneath our feet, and in none is speculation more dangerous; yet, apart from speculation, it is little that we can say regarding the constitution of the interior of the earth. We know, with sufficient accuracy for most purposes, its size and shape: we know that its mean density is about $5\frac{1}{2}$ times that of water, that the density must increase towards the centre, and that the temperature must be high, but beyond these facts little can be said to be known. Many theories of the earth have been propounded at different times: the central substance of the earth has been supposed to be fiery, fluid, solid, and gaseous in turn, till geologists have turned in despair from the subject, and become inclined to confine their attention to the outermost crust of the earth, leaving its centre as a playground for mathematicians.

The object of this paper is not to introduce another speculation, but to point out that the subject is, at least partly, removed from the realm of speculation into that of knowledge by the instrument of research which the modern seismograph has placed in our hands. Just as the spectroscope opened up a new astronomy by enabling the astronomer to determine some of the constituents of which distant stars are composed, so the seismograph, recording the unfelt motion of distant earthquakes, enables us to see into the earth and determine its nature with as great a certainty, up to a certain point, as if we could drive a tunnel through it and take samples of the matter passed through. The subject is yet in its infancy, and much may ultimately be expected of it; already some interesting and unexpected results have come out, which I propose to deal with in this paper.

So long ago as 1894 the late E. von Rebeur Paschwitz, recording the Japanese earthquake[1] of March 22nd, found that the record showed three separate disturbances or phases, but I believe that the true character of this threefold disturbance was not established until 1900, when I showed,[2] by a study of the available data, that the disturbance set up by a great earthquake was split up into three distinct forms of wave-motion, propagated at different rates and along different paths, giving rise to three distinct phases in its distant record. Of these, the third and latest was shown to be due to surface-waves, that is to say, wave-motion propagated along, or close to, the surface of the earth; but it was also shown that the other

[1] Peterm. Mitth. vol. xli (1895) pp. 13-21 & 39-40.
[2] Phil. Trans. Roy. Soc. ser. A, vol. cxciv (1900) pp. 135-74.

two phases, forming what are known as the preliminary tremors, represented the cropping-out of mass-waves which had travelled through the earth. It is these two phases alone with which I am at present concerned, for the third-phase waves can obviously give no information regarding the interior of the earth, as their wave-paths lie along its surface.

The researches of Dr. C. G. Knott[1] and Dr. P. Rudzki[2] have shown that no simple form of wave-motion can be transmitted through the heterogeneous rocks forming the outermost crust of the earth, and that the records from instruments situated near the origin of an earthquake cannot show any sorting-out of different kinds of wave-motion. It is only in more homogeneous material that this sorting-out can take place, and it is only at a distance of 10 degrees of arc, or about 700 miles, from the origin that the three-phase character of the record begins to appear. The waves emerging at this distance have evidently traversed more homogeneous material for a part of their course; in this part there has been a sorting-out of the forms of wave-motion into which the disturbance has been converted, and the fact that the sorting-out can be detected at so comparatively small a distance from the origin shows that the outer crust must be, comparatively, very thin. I have not been able to collect sufficient data for an accurate estimate of its thickness, but this cannot be more than about a score of miles,[3] and below it comes material of a very different character, which not only allows a sorting-out of different forms of wave-motion, but, as has been shown by Prof. Milne,[4] transmits these at a velocity much greater than is met with in the outer crust. If the figures given in the following pages do not bear out in detail his further conclusions regarding the homogeneity of the whole of the core and the rectilinear propagation of the wave-motion, this must be ascribed to the accumulation of more data than were available when he wrote. As will be seen, they are confirmed in essentials, so far as the outer six-tenths of the radius are concerned, and in the central four-tenths the first-phase waves, which alone were dealt with by Prof. Milne, are so little affected that the change might easily have escaped recognition but for the clue given by those of the second phase. It is, therefore, desirable to devote a little space to the demonstration of the reality of a distinction in kind between these two sets of waves.

In my paper quoted above, I pointed out that the different rates of propagation of the first and second phases showed that they must be referred to different forms of wave-motion, which I interpreted as being, probably, the two known forms—compressional and distortional—which can be transmitted by a homogeneous solid. As

[1] Trans. Seismol. Soc. Japan, vol. xii (1888) pp. 115–36.

[2] Beiträge zur Geophysik, vol. iii (1898) pp. 519–40.

[3] There is some, seismological, indication of a want of uniformity in this thickness, for earthquakes originating off the eastern coast of Japan exhibit a three-phase character at less distances from the origin than appears to be the case in Europe, indicating a lesser thickness of the outer crust in the former region.

[4] 'Nature' April 9th, 1903, vol. lxvii, pp. 538–39.

regards the first phase, the conclusion was only one which had already been suggested, and is still generally held; but, as regards the second, my interpretation has been traversed in two separate ways.

The first is by the Rev. O. Fisher, who, believing my interpretation to be inconsistent with his theory of the earth, has propounded a most ingenious explanation of these second-phase waves.[1] It does not seem to me that there is any insuperable incompatibility between Mr. Fisher's theory of a fluid centre and the hypothesis that the second-phase waves are distortional. We know nothing of the behaviour of matter exposed to the pressures prevailing in the interior of the earth, and it is not wholly inconceivable that a fluid under pressure of millions of atmospheres might be enabled to transmit the distortional waves which it is unable to transmit under pressures with which we are familiar. I do not, however, insist on this point, as it is immaterial to my present purpose: all that is material is that the wave-motion, in the first and second phases, differs essentially; and this is accepted by Mr. Fisher.

It is also indicated, apart from the arguments which I have already urged,[2] by the records of Prof. Vicentini's type of seismograph, composed of two heavy masses, one free to move horizontally, the other free to move vertically. In the records of great earthquakes originating at a distance of 90° of arc or more, it is found that the former gives a very small displacement for the first phase, while the latter frequently registers the maximum displacement of the whole disturbance. In the second phase the conditions are reversed, and, while the mass free to move vertically seldom gives any indication of disturbance, that which is free to move horizontally gives a very large displacement. This difference in the character of the record in the two phases shows that the movement is different, and incidentally tends to support the interpretation that I have proposed: for, if the first phase represents the outcrop of a disturbance transmitted through the earth as a condensational wave, then vertical movement would preponderate over horizontal at distances of 90° or more; while, if the second phase is caused by distortional waves, horizontal movement should preponderate in it.

This difference in the character of the records of the two phases may also be used as an argument against the idea which has been adopted in Japan,[3] that the first and second phases represent wave-motion of similar character, transmitted at different rates through layers at different depths from the surface, but in both cases parallel to, and at no great depth below, it. As this contention is incompatible with the figures given below, it need not be dealt with in this place, and the facts may be left to speak for themselves.

[1] 'On the Transmission of Earthquake-Waves through the Earth' Proc. Cambridge Phil. Soc. vol. xii (1903-1904) pp. 354-61.

[2] Phil. Trans. Roy. Soc. ser. A, vol. cxciv (1900) pp. 162-66.

[3] A. Imamura, Publications of the Earthquake-Investigation Committee in Foreign Languages, No. 16, Tokio, 1904 and later issues *passim*.

II. The Data.

In dealing with the data, it is necessary to make some selection from the large amount of material which has been collected, and to confine our attention to those records in which accuracy is *prima-facie* probable; this limits us to those earthquakes the place and time of origin of which can be determined with accuracy, and which were also of sufficient magnitude to give complete records on distant seismographs. This last reservation is necessary, for many earthquakes of great local severity are only imperfectly recorded at a distance of even a quarter of the circumference of the globe, and the portion lost is always that of the preliminary tremors.

These limitations leave only fourteen disturbances for consideration, some of which consisted of two or three distinct earthquakes, starting from the same origin at short intervals from each other. Of these, details have been published in a collected form in some cases only, in the others they are still in manuscript; but those that have been published will serve to show the manner in which scattered details are grouped and dealt with. The earthquakes utilized in this paper are:—

1. Japan, March 22nd, 1894; $10^h\ 22{\cdot}5^m$, 43°·0 N., 146°·0 E. Phil. Trans. Roy. Soc. ser. A, vol. cxciv (1900) pp. 139–40, quoting Peterm. Mitth. vol. xli (1895) pp. 14–21.
2. Argentine, Oct. 27th, 1894; $20^h\ 55{\cdot}5^m$, 28°·5 S., 69°·0 W. *Ibid.* pp. 140–142.
3. Japan, June 15th, 1895; three shocks, $10^h\ 31{\cdot}0^m$, $19^h\ 3{\cdot}2^m$, $22^h\ 58{\cdot}0^m$, 39°·5 N., 144°·5 E. *Ibid.* pp. 142–145, with some further details from Beiträge zur Geophysik, vol. vi (1903–1904) p. 408.
4. Japan, Aug. 31st, 1896; $8^h\ 7{\cdot}1^m$, 39°·7 N., 140°·8 E. *Ibid.* pp. 145–47.
5. India, June 12th, 1897; $11^h\ 5{\cdot}0^m$, 26° 0 N., 91°·0 E. *Ibid.* pp. 147–49.
6. Japan, Aug. 5th, 1897; $0^h\ 9{\cdot}4^m$, 39°·5 N., 144°·5 E. *Ibid.* pp. 149–51.
7. Turkestan, September 17th, 1897; two shocks, $15^h\ 28{\cdot}0^m$, $17^h\ 36{\cdot}0^m$, 39°·0 N., 68°·0 E. *Ibid.* pp. 151–55.
8. Japan, April 22nd, 1898; $23^h\ 34{\cdot}2^m$, 39°·5 N., 143°·0 E. *MS.*
9. Japan, August 9th, 1901; two shocks, $9^h\ 23{\cdot}5^m$ and $18^h\ 33{\cdot}5^m$, 40°·5 N., 141°·5 E. *MS.*
10. Philippines, Dec 14th, 1901; $22^h\ 57{\cdot}5^m$, 13°·5 N., 121°·25 E. *MS.*
11. Guatemala, April 19th, 1902; $2^h\ 22{\cdot}0^m$, 14°·5 N., 91°·25 W. Proc. Roy. Soc. ser. A, vol. lxxvi (1905) pp. 102–111.
12. Kashgar, August 22nd, 1902; $3^h\ 1{\cdot}0^m$, 39°·5 N., 75°·9 E. *MS.*

To which may be added two earthquakes the time of origin of which is only known by inference from distant records, as mentioned lower down:—

13. Alaska, September 4th, 1899, $0^h\ 20{\cdot}5^m$; September 10th, $17^h\ 1{\cdot}5^m$ and $21^h\ 39{\cdot}5^m$; about 59°·5 N., 140°·0 W. *MS.*
14. Ceram, September 29th, 1899, $17^h\ 3{\cdot}0^m$, 3°·5 S, 128°·5 E. *MS.*

In dealing with the records, it is necessary to bear in mind that they are liable to certain errors. In the first place, many earthquakes consist, not of a single impulse only, but of two or more, separated from each other by intervals of some minutes; and it is not uncommon for the disturbance due to the first impulse to be overlooked, either because it fails to overcome the inertia of the instrument, or because the disturbance is too small to be recognized.

Secondly the disturbance, instead of beginning abruptly, as is sometimes the case, may come in gradually; and when this is the case, it is easy for the times of commencement of each phase to vary by a minute or more, on the records of different instruments, or even in the reading of the same record by different individuals. Either of these causes will make the recorded time late, but it also happens, not commonly, though often enough for the contingency to be borne in mind, that one station or a group of stations is affected by some small local disturbance, which almost coincides with a distant earthquake, and leads to the apparent commencement being too early. Apart from these sources of error, there is also that which may easily occur in determining the time of origin of the earthquake: this will introduce an error into all the intervals, which is constant for each earthquake but varies for different ones both in amount and direction; it will, consequently, be eliminated when an average from a sufficiently-large number of earthquakes is taken, and will be partly eliminated even with the few which are at present available. The other sources of error are partly eliminated by averaging, but it is necessary to reject any records which are abnormally early or late, and to take only those which, by their close concordance with each other, show that they refer to the same phase of wave-motion. The average so obtained will naturally incline to be a little late, but is likely to be nearer the truth than any individual record, taken at random, and for this reason I shall deal, as far as possible, with averages rather than with single observations.

These averages may be obtained in two ways. In Table I (p. 472) are given all the group-averages that I have obtained: that is to say, averages of the records of groups of stations and instruments, each average being that of observations of a single earthquake, each group consisting of at least five distinct records from distances differing by less than five degrees of arc from each other.

In Table II (p. 473) a different treatment is used. The whole series of records from all earthquakes, excepting those numbered 13 and 14, were tabulated, and the average, of each group covering 5° of arc from the origin, taken. In this way we get a group of averages for each 5° from 45° to 95°, which are on the whole better than those in Table I.

For distances of 100° or over the possibility of averages is small, and I have given in Table III (p. 473) all the available records of first and second phases. The figures given are those originally determined by me, those enclosed in parentheses are times which, from their discordance with other records, were evidently misinterpreted or are otherwise doubtful.

It is evident that, having prepared these tables, we are no longer confined to the consideration of those earthquakes of which both the time and the place of origin are known; for, if the latter be known with even approximate accuracy, we can determine the former from distant records. This has enabled me to utilize two other disturbances for filling in gaps in the series of records.

The first of these disturbances is the group of earthquakes which originated in September, 1899, off the coast of Alaska. The times of origin can be determined from the records of the Italian Observatories, which lay at distances of from 73° to 81°, and the only records worth considering here are those from Cape Town, at a distance of about 150° from the origin. In all of them the commencement is almost imperceptible, and the recorded times, as compared with the times of origin, show that it was too late to represent the first phase of the original impulse, except possibly in the case of the third of these shocks, which gives an interval of 21·5 minutes. The second phase is well marked on all the records, and the times, as determined by me, on photographic copies of the original records, give intervals of 44·6, 45·7, and 45·5 minutes respectively[1]; the true interval, therefore, may be taken as about 45 minutes or a little more.

The second is the Ceram earthquake of September 29th, 1899. The place of origin of this earthquake can be fixed with a great degree of accuracy from Dr. Verbeek's description,[2] as close to 3°·5 S. lat., 128°·5 E. long. The time is not accurately known from local observatories; but the earthquake was well recorded by distant seismographs, and the records have been collected by Dr. E. Rudolph.[3] At Batavia, 22°·4 from the origin, the commencement was at 17^h $7{\cdot}3^m$, and at Calcutta, 46°·4 distant, the time was 17^h $11{\cdot}4^m$. Making the allowances indicated by Table I (p. 472), these give the time of origin as 17^h $3{\cdot}1^m$ and 17^h $3{\cdot}2^m$ respectively, which might be taken as near the true time of origin; but it will be noticed that the average-curve drawn on fig. 1 (p. 462) makes the averages just used to be about a minute and a half too early. The difference is due to one or other of the causes noticed above; and it will be better to adopt the intervals indicated by the time-curve, and accept 17^h $1{\cdot}5^m$ G.M.T. as the probable time of origin.

Accepting this as the time of origin, we get a group-average of observations from stations at distances of 103°·4 to 112°·2, the mean value being 110°·3; the mean interval for the first phase, from six records, is 17·1 minutes, and for the second phase, for ten records, is 27·3 minutes. At Cordoba (Argentina), 143° from the origin, the record as published in the British Association Seismological Circulars, gives the commencement at 17^h $21{\cdot}8^m$ and the maximum at 17^h $44{\cdot}3^m$ Greenwich time. A tracing of the record shows that this belongs to the second phase, which is well marked, and commenced about ·5 min. earlier. This gives intervals from the origin of 20·3 minutes for the first, and 42·3 minutes for the second phase.

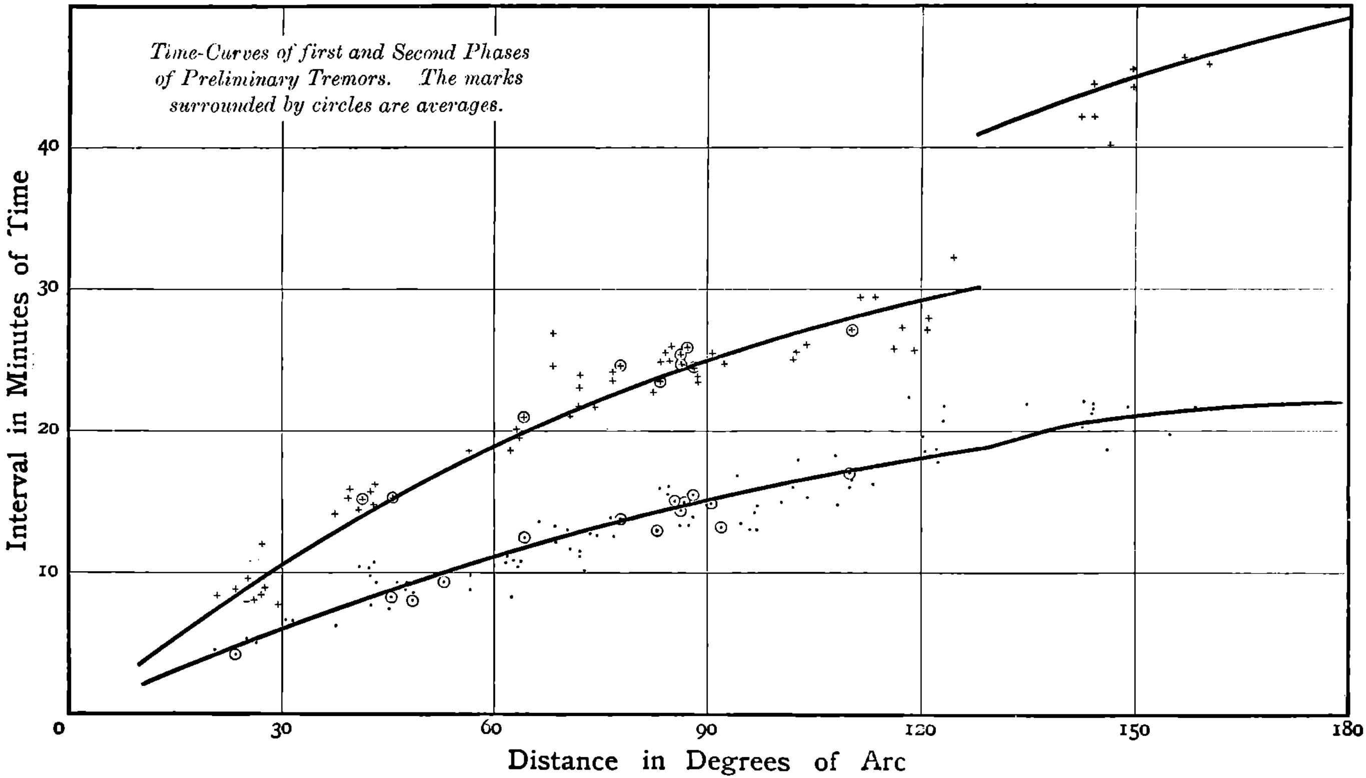
Such are the materials available. As may be noticed, there are discrepancies, and the time-intervals do not increase regularly with

[1] In the first case, the time is a little uncertain, owing to failure of the occulting watch. See Brit. Assoc. Seismological Circular, No. 1, 1900.

[2] 'Kort Verslag over de Aard- en Zeebeving op Ceram den 30 Sept. 1899' Natuurkundig Tijdschrift voor Nederlandsch-Indië, vol. lx (1900) pp. 219–28.

[3] Beiträge zur Geophysik, vol. vi (1904) pp. 238–66.

Fig. 1.


Time-Curves of first and Second Phases
of Preliminary Tremors. The marks
surrounded by circles are averages.
Interval in Minutes of Time
10
20
30
40
0
30
60
90
120
150
180
Distance in Degrees of Arc

the distance: the discrepancies being due, as has been explained, partly to inaccuracies in the distant records, and partly to errors in determining the time of origin. Another possible source of discrepancy is the possibility that the rate of propagation is not uniform in every direction, and that the time taken by wave-motion in travelling, say from Japan to Europe, is different from that taken by the same form of wave-motion in travelling from an equal distance in America. There are some indications that such is the case; but the difference is small, in comparison with the whole interval, and as the point is not material to the present investigation, it may be ignored, and the irregularities smoothed to a regular time-curve.

This is best done graphically, as is represented in fig. 1 (p. 462), where the averages of groups, and the single observations not adapted to averaging, have been plotted, and average time-curves drawn for the first and second phases.

It will be seen that the time-curves of the first two phases are very similar in shape, up to 120° from the origin; but beyond this they differ radically in form. That of the first phase, after an irregularity between 130° and 140°, becomes very flat and proceeds almost horizontally from 150° to 180°; that of the second phase comes to an end at 130° from the origin, and is continued some 11 minutes farther up. It is the explanation of these irregularities with which this paper is mainly concerned; and, to simplify the consideration of this problem, I have found it convenient to tabulate the figures represented by the time-curves, as they are more concordant and useful than the individual records. This has been done in the appended table (p. 464), which gives the intervals of time, to the nearest even minute, at each 30° of arc from the origin to its antipodes, the value for the last-named being inferred and not the result of direct observation.

In addition to the intervals of time, the mean apparent rate of propagation along the arc and the chord is given, regarding which a few words of explanation are requisite. The difference between the true and the apparent rate of propagation of an earthquake has long been familiar to seismologists. The first is the rate as measured along the direction in which the wave-motion is propagated, the latter is the resulting apparent rate of propagation, measured in some other direction, usually along the surface of the earth. Neither of these is readily determinable at any point, except by the construction of the time-curves, such as are drawn in fig. 1 (p. 462); but if the time be known at two points, then the difference in distance from the origin, divided by the difference in time, gives the mean apparent rate of propagation as between those points. In the table (p. 464) one point is supposed to be the origin, and two values of the mean apparent rate of propagation in kilometres per second are given, according as the distance is reckoned along the surface of the earth or in a straight line through it. It is not suggested that the wave-paths lie along either of these lines, but the values calculated are the material from which the true form of the wave-paths can be determined.

Table showing the Time taken by Waves of the Two Phases of Preliminary Tremors in travelling from the Origin, and Mean Apparent Rates of Transmission along Arc and Chord; together with Distances, and Maximum Depth of Chord from Surface, calculated for a Spherical Globe of 40,000 kilometres circumference, and for Intervals of 30° of arc.

Distance.	First Phase.			Second Phase.			Length		Maximum depth of Chord.
	Interval.	Rate along		Interval.	Rate along		of Arc.	of Chord.	
		Arc.	Chord.		Arc.	Chord.			
Degrees.	Minutes.	Km. sec.	Km. sec.	Min.	Km. sec.	Km. sec.	Km.	Km.	Radius.
30	6	9·26	9·15	11	5·05	4·99	3,333·3	3,295·5	·034
60	11	10·10	9·65	19	5·85	5·58	6,666·6	6,365·5	·134
90	15	11·11	10·00	25	6·66	6·00	10,000·0	9,003·5	·293
120	18	12·35	10·21	29	7·66	6·34	13,333·3	11,027·5	·500
150	21	13·23	9·76	45	6·17	4·55	16,666·6	12,297·0	·741
180	22	15·15	9·65	50	6·67	4·24	20,000·0	12,733·0	1·000

It should be noted that the intervals and resulting rates must not be taken too literally: the result of averaging observations is, indubitably, to increase the interval and lessen the apparent rates; but, besides this, an allowance ought to be made for the reduced rate of propagation of the disturbance through the outer crust of the earth. The amount of these corrections is not accurately determinable, but as they are in the same direction in every case, and as both together would, probably, not amount to a minute of time, they may be neglected so far as the conclusions drawn below are concerned.

III. The Deductions.

Wave-motion originating at any point in the earth will be propagated in all directions from it, and whatever the nature of these waves their wave-paths will be straight lines so long as the velocity of propagation remains constant; but, if this varies, the course of the wave-paths will be altered according to the laws of refraction, which are to be found in every text-book of physics. These laws hold good whatever be the nature of the wave-motion, although, in the case of elastic waves, the rate of propagation is dependent on two factors—the elasticity and density of the medium through which they are propagated. From this it will be seen that any information that we can get regarding the form of the wave-paths will indicate the changes, if any, in the rate of propagation, and thence in the physical condition, of different parts of the earth traversed by the wave-paths which emerge at different parts of its surface.

It will not be necessary to enter into details as to the manner in which the wave-paths can be determined from observations of the time of arrival of the disturbance; for the subject has been fully dealt with as a mathematical problem by Dr. Rudzki [1] of Cracow, and it will only be necessary to apply his conclusions.

In the first place, if waves are propagated along the surface of the earth, or at a short distance below but parallel to it, the mean apparent rate of propagation, as measured along the surface of the earth, will be constant for all distances. This is the case, or nearly the case, for the third-phase waves, which are, consequently, accepted as surface-waves, and can, therefore, give no information regarding the central portions of the earth. In the first and second phases this is obviously not the case: in the first phase there is a continuous increase in the apparent rate of propagation, and although there is irregularity in the rates calculated for the second phase, these are higher for distances beyond 90° than for lesser distances. These facts lead to the conclusion that the first and second-phase waves cannot be surface-waves, nor waves propagated a short distance below the surface, but must be mass-waves propagated through the body of the earth.

In considering the form of the paths along which these waves are propagated, it will be convenient to consider each quadrant

[1] Beiträge zur Geophysik, vol. iii (1898) pp. 495–518.

separately, dealing first with the wave-paths which emerge at distances up to 90° from the origin, and then with those which emerge at distances beyond that.

If the rate of propagation through the earth were uniform in all directions and at all depths, the wave-paths would be straight lines, and the mean apparent rate of transmission, along the chord, would be the same for all distances. It is obvious that this is not so, for the apparent rate of propagation, as measured along the chord, increases continuously up to 90°, in the case of both first and second phase-waves. This means that the waves travel faster as they penetrate to greater depths, and consequently the wave-paths are not straight lines, but curves whose convexity is directed towards the centre. Besides this, the fact that the increase in apparent rate of propagation is proportionately greater in the case of the second phase, shows that the curvature of its wave-paths is greater than in the case of the first-phase waves.

It must, however, be noticed that, although the increase in apparent rate of propagation is greater in the one case than in the other, yet the rate of increase as between 30°, 60°, and 90° is practically the same in both cases. The actual figures are as follows :—

		First Phase.	*Second Phase.*
Increase of apparent velocity along the chord	30° to 60°	·055	·118
Ditto	60° to 90°	·036	·075
Ratio of increments............	......	·65	·64

These figures show that the apparent rate of propagation increases with the distance about twice as rapidly in the case of the second as in that of the first-phase waves; that the rate of increase is not uniform, but diminishing with increasing distance; and that in both cases this alteration is in the same direction and at the same rate. This suggests that the increase in rate of propagation with increase in depth of the wave-paths is not due to their passage through material of a different character, but may be merely the effect of increased pressure and temperature, and consequently, that the substance of which the earth is composed—below the outer crust—undergoes no material change in composition or physical condition, at least to the depths reached by the wave-paths of earthquake-waves emerging at 90° from the origin.

This much is independent of any assumption regarding the nature of the wave-motion, but without making some assumption regarding this, no further information is attainable. I shall take it that the first-phase waves are condensational—this being generally acknowledged—and that the second-phase waves are distortional, an assumption which I regard as more than probable, and on these assumptions it is possible to estimate the proportion which the modulus of rigidity bears to the bulk-modulus, or resistance to compression. In making this estimate we may take the wave-paths for the two waves as being so nearly coincident that there is no material difference in the density of the medium. The calculation being simple, it is only necessary to state the results, which are that on wave-paths emerging at 30° from the origin the rigidity is ·385 of the bulk-modulus, for paths emerging at 60° it is ·446, and for those emerging at 90° it is ·493; that is to say, the rigidity or power of resisting distortion increases at a greater rate than the solidity, or power of resisting compression.

If absolute values of these two moduli are required, it is necessary to make some assumption regarding the density of the medium through which the waves are transmitted, and if it be assumed that Laplace's law of densities is correct and that the mean density [1] of the medium traversed is about the same as that at the greatest depth reached by the chord, we get the following values for the mean rigidity and bulk-modulus, both being measured in C.G.S. units:—

		Modulus of Resistance to	
Arc.	*Assumed density.*	*Compression.*	*Distortion.*
30°	3·00	$151{\cdot}6 \times 10^{10}$	$74{\cdot}7 \times 10^{10}$
60°	4·25	$219{\cdot}4 \times 10^{10}$	$132{\cdot}3 \times 10^{10}$
90°	6·20	$322{\cdot}4 \times 10^{10}$	$223{\cdot}2 \times 10^{10}$

These figures should be regarded as arithmetical curiosities rather than actual measures, for, apart from uncertainty regarding the density of the medium, the mean apparent rate of propagation, as measured along the chord, is certainly less than the true mean rate, as measured along the actual wave-path, and the maximum rate is greater than this again; yet, despite this, the figures indicate that the material traversed by the waves is endowed with a very high degree of rigidity and resistance to compression. In the case of the waves emerging at 90° from the origin, the material traversed has, on the average, nearly 12 times the resistance of granite to compression and 15 times its rigidity; if the density remains constant, these figures would be reduced by about three-tenths, but on the other hand the maximum values will be higher.

It must, however, be borne in mind that this high degree of rigidity, as against stresses of very short duration, is quite compatible with the yielding to stresses of long duration, which is required by known facts of structural geology, and need not necessarily be inconsistent with those movements, of the nature of convection-currents, which Mr. Fisher [2] believes to exist in the interior of the earth.

Turning now to the second quadrant, it will be convenient to take each phase separately, and to commence with the second phase.

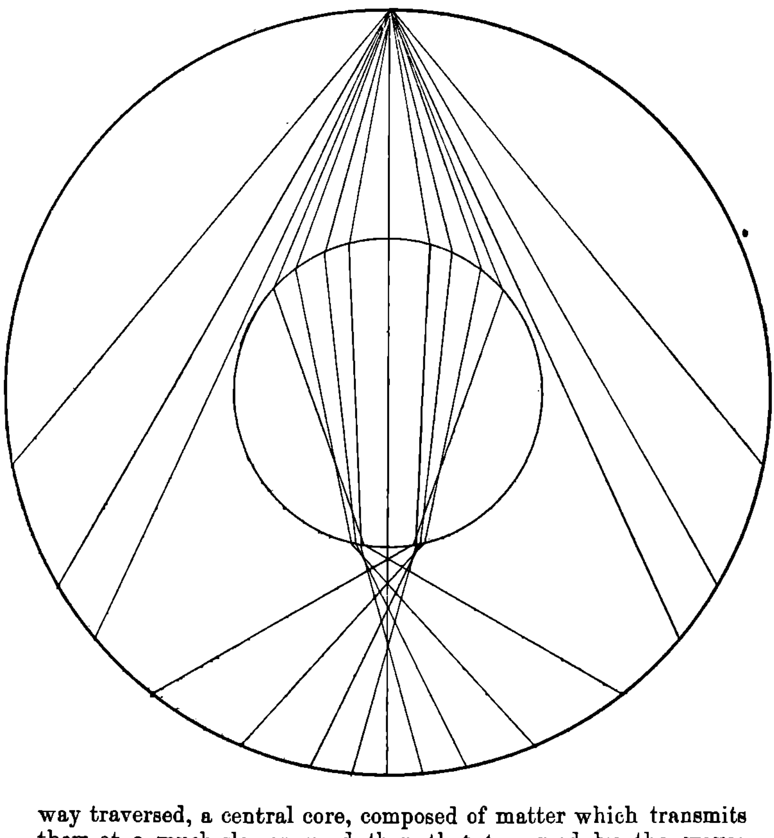
The table (p. 464) shows that at 120° the increase in the mean apparent rate of propagation is more than maintained, but too much importance must not be attached to the exact figures, for the interval at 120° is somewhat uncertain. Most of the records from about this distance are late commencements, attributed to the second

[1] Strictly speaking, the square root of the mean of the squares of densities.
[2] 'Physics of the Earth's Crust' 2nd ed. (1889) chapters vi & xxiii.

phase; and, if these be excluded, the interval will be a little longer, and the apparent rate of propagation a little less, than is indicated in the table.

At 150° from the origin we find a remarkable decrease in the mean apparent rate of propagation, which drops from an average of over 6 to about $4\frac{1}{2}$ kilometres per second, and the most obvious explanation of the decrease is that these waves, penetrating to greater depths, have entered, and for part of their

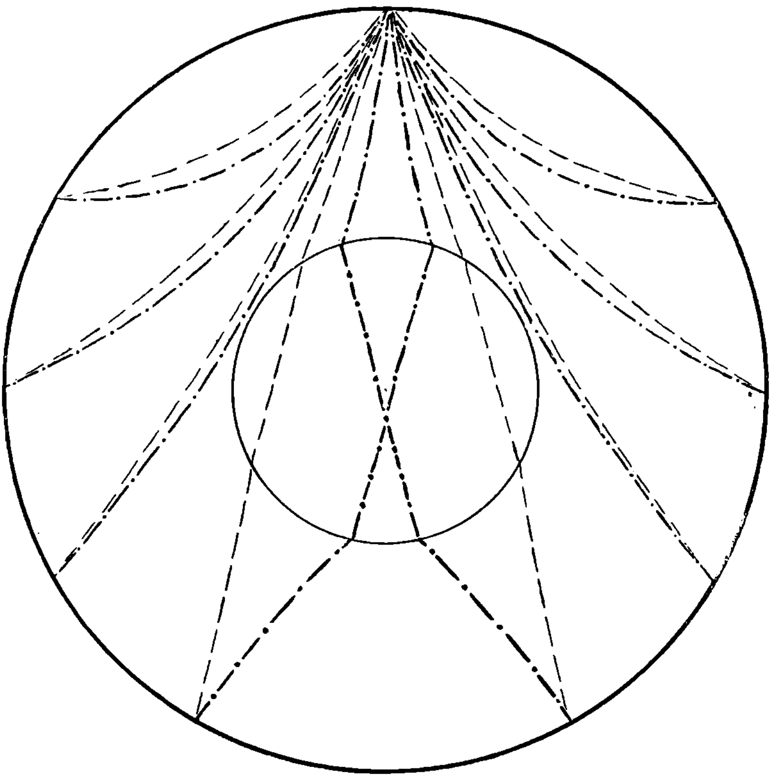
Fig. 2.



way traversed, a central core, composed of matter which transmits them at a much slower speed than that traversed by the waves emerging at lesser distances from the origin. The only other alternative is that the time-interval is wrong, and that we are not dealing with the second-phase waves at all.

As regards this hypothesis, I may point out that all the determinations used at distances of over 120° are derived from my own examination of the original records or copies from them. In every case the second phase, as adopted, presents the same characters as those which I had recognized at lesser distances; and if the times given do not refer to the second phase (in the sense used elsewhere by me), then this phase is not represented at all in the more distant records, and instead of a central core which transmits the waves more slowly, there must be one which is incapable of transmitting them at all, thus leading to the same conclusion, that

Fig. 3.



[The broken lines represent the first phase, the broken-and-dotted lines the second phase, ~~and the continuous curve the third phase.]~~

the deeply penetrating wave-paths enter matter of very different constitution from that traversed by the shallower paths.

Rejecting the supposition that the second-phase waves are extinguished by the central core, and accepting the more probable one that the rate of transmission is reduced in it, there remain two important questions to be answered, namely, the size of the core and the rate of transmission of the waves in it.

As regards the size of the core, we have seen that it is not penetrated by the wave-paths which emerge at 120°; and the great decrease at 150° shows that the wave-paths emerging at this distance have penetrated deeply into it. Now, the chord of 120° reaches a maximum depth from the surface of half the radius, and we have seen that the wave-paths up to this distance are convex towards the centre of the earth, so it may be taken that the central core does not extend beyond about ·4 of the radius from the centre.

As regards the rate of transmission of the waves, the data hardly deserve elaborate mathematical treatment until more have been collected, but it is easy to arrive at an approximate estimate of the rate of transmission and the nature of the wave-paths. The chord of 150° has a length of 12,297 kilometres, of which 8413 km. lies in the outer ·6 of the radius and, at a mean rate of 6 km. sec., requires 23·4 minutes, leaving 21·6 minutes for the remaining 3884 km., or a mean rate of 3 kilometres per second.

This reduction in speed has an important and unexpected result, for it means a refractive index of 2·0 and a great deviation of the wave-paths as they enter the central core. As a first approximation to the actual course of the wave-paths, I give, in fig. 2 (p. 468), a representation of what they would be on the supposition of a central core, occupying ·4 of the radius, in which the rate of propagation is one half of that in the outer shell; in this it will be seen that the wave-paths emerging at 150° reach their emergence after passing on the opposite side of the centre of the earth, and exhibit that concavity towards the centre which Dr. Rudzki's investigation requires where increase in depth of wave-path is accompanied by a decrease in the rate of propagation.[1] The actual wave-paths, however, are not, as has been shown, composed of straight lines, and the real wave-paths must be more like what is indicated in fig. 3 (p. 469), which may be taken as correct in kind, though perhaps wrong in detail, as to the actual position of the wave-paths.

The high index of refraction prevents the formation of a complete shadow-band, for the most extreme of the rays which enter the central core suffer so great a deviation that their point of emergence at the surface overlaps that of the last rays which miss the central core; but an inspection of figs. 2 & 3 will show that there should

[1] The wave-paths shown in fig. 2 give, for an emergence at 150° and at a time-interval of 45 minutes, a rate of transmission of 3·5 km. sec. in the central core, and 7·0 km. sec. in the outer shell. These values are higher than can be admitted; the explanation probably lies in the shortening of these long-distance wave-paths which results from their curvature, as shown in fig. 3, and possibly also in a lesser ratio than that of 2 : 1 of the rates of transmission, or a lesser size of the central core. It may also be noticed that rates of 7·0 and 3·5 km. sec. respectively give an interval for the diameter of about 42·5 minutes; and although this value cannot be accepted, it indicates a possibility that the emergence of the second-phase waves at the antipodes of the origin may actually be earlier than at a distance of 150°.

be a zone, at about 140° from the origin, where the second-phase waves would be so dispersed, and consequently feeble, that it would practically amount to a shadow, and the second phase should be absent in records from this distance, or much more feebly marked than in those from greater or lesser distances.

The effect will be modified by the fact that the transition from central core to outer shell is not abrupt but gradual, though comparatively rapid; yet it is worth noting that, so far as the limited amount of available material may be trusted, the second phase is certainly much less marked at about 140° from the origin than at distances of less than 130° or more than 150°. For instance, the Guatemala earthquake was recorded at Bombay, 144° from the origin, by three instruments. On one record no indication of a second phase can be found; on another it is so indistinctly marked that it can hardly be recognized as such; and even on the Milne pendulum, which shows what I take to be the second phase most distinctly, it is not at all characteristic. The Batavian record of this same earthquake, at a distance of 160° from the origin, shows it much more distinctly; and on the Cape-Town records of the Alaskan earthquakes at 150° from the origin it is easily recognizable.

These considerations lead to the conclusion that the time-curve of the second-phase waves is not a continuous line. Up to about 130° it is continuous, and represents the emergence of waves which have travelled directly from the origin; beyond that distance it represents waves which have been refracted, after passing on the opposite side of the centre of the earth, and it would be misleading to join the two into one continuous curve. For these reasons, the second-phase time-curve has been drawn as it is shown in fig. 1 (p. 462).

We may now turn to the first-phase waves, and see how they are affected by the central core. At 120° the increase in mean apparent rate of transmission is maintained, but at 150° the rate has dropped to 9·76 km. sec., and the value is a good one. There can be no doubt that the drop is real, but it is much less than in the case of the second-phase waves, and merely represents a diminution of the rate of propagation by about one-tenth.

From this it will be seen that the central core behaves differently from the outer shell with regard to the first, as with regard to the second phase, but the change is much less in amount, and would probably have remained undetected were it not for the very conspicuous alteration in the case of the second-phase waves.

IV. Conclusions.

From the considerations detailed in the foregoing pages, I conclude that the interior of the earth, after the outermost crust of heterogeneous rock is passed, consists of a uniform material, capable of transmitting wave-motion of two different types at different rates of propagation: that this material undergoes no material change in physical character to a depth of about six-tenths of the radius, such change as takes place being gradual and probably accounted for sufficiently by the increase of pressure; and that the central four-tenths ot the radius are occupied by matter possessing radically-different physical properties, inasmuch as the rate of propagation of the first phase is but slightly reduced, while the second-phase waves are either not transmitted at all, or, more probably, transmitted at about half the rate which prevails in the outer shell.

If these waves are to be explained as those of condensation and distortion, then the ratio between the modulus of rigidity and the bulk-modulus is only two-thirds of that obtaining in the outer shell; but whether this interpretation be adopted, or that of Mr. Fisher, or some other yet unproposed, we still have a central core the behaviour of which with regard to these waves differs materially from that of the outer shell. I do not propose to enter into speculative grounds, or to offer any opinion as to whether this central core is composed of iron, surrounded by a stony shell, or whether it is the central gaseous nucleus of others. On this occasion, it is enough to have shown that there is a difference which cannot be overlooked, and must be taken into account in any hypothesis that may be formed regarding the constitution of the interior of the earth.

Table I.

Group-Average Intervals for Propagation of First- and Second-Phase Waves from their Origin to the Place of Observation.

Distance: Degrees.	*First Phase.*		*Second Phase.*		*Earthquake. Ser. No.*
	Observ. No.	*Interval: Minutes.*	*Observ. No.*	*Interval: Minutes.*	
23·4	6	4·2	...	...	12
41·7	...	...	8	15·0	7
45·9	17	8·2	7	15·1	12
48·5	5	8·0	...	...	"
53·1	5	9·1	...	...	"
64·2	13	12·4	11	20·7	5
78·0	7	13·5	5	24·4	11
83·3	33	12·8	30	23·5	9
85·2	5	14·8	...	...	1
86·3	5	14·1	8	24·6	4
86·8	7	14·5	10	25·2	8
87·9	...	...	6	25·7	3
88·1	9	15·1	7	24·4	6
90·8	20	14·8	14	25·1	11
92·3	11	13·1	8	24·6	10
145·6	5	21·7	...	...	11

Table II.

Average for all Earthquakes, grouped for intervals of 5° of arc.

Distance: Degrees.	First Phase.		Second Phase.	
	Observ. No.	Interval: Minutes.	Observ. No.	Interval: Minutes.
45	14	8·3	8	15·0
50	8	8·4	...	...
55	7	9·2	...	...
65	13	11·8	11	19·8
70	9	11·9	9	22·9
75	8	13·1	8	23·6
80	13	12·7	10	23·8
85	42	13·6	47	24·1
90	36	15·0	37	24·9
95	14	13·9	12	25·3

Table III.

Intervals from Origin to Commencement of First and Second Phases, as recorded at Distances of over 100° from the Origin.

Distance.	Earthquake No. and Place of Record.		First Phase.	Second Phase.
100·3	11	Nikolaiew	15·0	...
102·5	11	Wellington (N.Z.)	16·0	25·0
102·8	2	Rome	17·8	25·2
104·5	11	Christchurch (N.Z.)	15·2	26·0
108·5	10	San Fernando	14·7	...
108·5	10	Cape Town	18·1	27·6
110·7	11	Tiflis	16·2	...
”	”	”	16·2	...
”	”	”	16·3	...
111·9	11	Irkutsk	...	29·5
113·9	11	Cape Town	16·4	29·6
117·6	2	Nikolaiew	...	27·1
118·4	10	Toronto	(22·6)	...
120·2	12	Christchurch (N.Z.)	19·6	(40·2)
120·9	2	Kharkow	18·6	...
121·3	11	Tashkent	17·9	27·2
”	”	”	18·5	28·1
123·3	10	Baltimore	21·7	...
124·8	9	Trinidad	...	32·5
134·9	9	Cape Town	22·0	...
”	”	”	24·1	...
142·9	11	Calcutta	22·0	...
144·1	11	Bombay	21·3	(42·5)
”	”	”	21·8	(44·8)
”	”	”	21·4	...
146·8	12	Cordoba (Arg.)	18·5	40·2
149·8	11	Perth (W.A.)	21·8	(40·0)
154·4	10	Trinidad	21·5	...
157·4	9	Cordoba (Arg.)	19·8	(39·5)
”	”	”	23·6	46·5
160·4	11	Batavia	21·7	46·0