Laboratory Calibration of Gravimeters

P. Varga Budapest

INTRODUCTION

Present report is a continuation of the paper on the research work carried out in Hungary for the determination of the scale values of gravimeters presented during 4. Internationale Alpengravimetrie-Kolloquium (Barta G., Hajósy A., Varga P., 1988).

The motivation of our work was:

1. for the better understanding the earth tidal parameters obtained from the gravimetric records we need an absolute calibration possibility which has an external accuracy equal (or better) to 0.1 %.
2. during last years a lot of problems aroses in connection with the three hundred years old law of gravity. To investigate a part of this problems we have to compare scale determinations of gravimeters carried out on different ways. Doing this comparison we must be extremely cautious because the small magnitude gravity variations used for this study.

A. NEED OF THE IMPROVEMENT OF THE RELIABILITY OF THE GRAVITY EARTH TIDE OBSERVATIONS

It was mentioned by many authors (see for example Molodensky and Kramer, 1980) that gravimetric body tide anomalies can be used for the study of lateral heterogeneities within the Earth. For this purpose we must consider the realistic magnitude of the tidal number variations over the Earth's surface. For this purpose, we used the works of Woodhouse and Dziewonski (1984) and of Dziewonski (1984), which carried out the mapping of the upper and the lower mantles, respectively. The investigation of the upper mantle structure was based on the shear wave velocities (Vs), whereas the structure of the lower mantle was infered from the compressional wave velocity (Vp) data. In the paper of Woodhouse and Dziewonski (1984), three-dimensional modelling of the Earth's structure was performed by inversion of seismic waveform till a pepth of 670 km (r/a=0.90 relative earth radius, where r is the distance from the centre of the Earth, a=6371 km). The sizes of the speed anomalies of the Vs waves were ± 8% at a depth of 50 km, ± 2.5 km at 250 % and ±2 % at 650 km. These lateral velocity anomalies are surprisingly big and they are comparable with the velocity jumps across the radial structural discontinuities in the upper mantle. For instance in PREM (Preliminary Reference Earth Model) the velocity jumps are 15 % at the Mohorovicic discontinuity surface, 6 % at the depth 220 km, 3 % at, 400 km and 7 % at 670 (the depth of the transitional zone C between the upper and the lower mantles Dziewonski (1984) carried out the three dimensional mapping of the lower mantle by means of Vp velocities The velocity anomalies at the top of the lower mantle reach about ± 3 %. At the core-mantle boundary (CMB) the size of the anomalies is the same. In the central parts of the lower mantle the velocity anomalies are slightly smaller (± 1 %).

If we introduce a simplification and suppose the equality of the Lamé parameters in the whole mantle we have $Vp = \sqrt{3}\ Vs$ and thus we can estimate lateral variations of both the compressional and shear wave velocities both in the upper and in the lower mantle. In this way we solved the sixth order differential equation system which describes elastic deformations of a spherically symmetric Earth with a liquid core to obtain the Love-Shida numbers (h, k, l) and their simple combination 1+h-3/2k used to describe the tidal gravity variations (Varga, 1987; Denis, Gerstenecker, Varga, 1987; Varga, Denis 1989). To evaluate the possible amount of variation of the gravity earth tidal combination 1+h-3/2k with respect to the reference values obtained for the PREM (1+h-3/2=1.1554) we used a slightly simplified version of the mapping of the Earth's mantle by Woodhouse and Dziewonski (1984) and Dziewonski (1984) (*Model A in Table 1*).

Table 1: Description of the models used for model calculations

Model A (Vp and Vs denote the original seismic speed values of PREM.

1.00>	*r/a*	*>*	*0.90*	*Vp=1.08*	*Vp*	*Vs=1.05*	*Vs*
0.90>	*r/a*	*>*	*0.85*	*Vp=1.03*	*Vp*	*Vs=1.02*	*Vs*
0.85>	*r/a*	*>*	*0.60*	*Vp=1.01*	*Vp*	*Vs=1.01*	*Vs*
0.60>	*r/a*	*>*	*0.55*	*Vp=1.03*	*Vp*	*Vs=1.02*	*Vs*

Model B The same as model A, but the density is increased by 2 % every where in the mantle.

For this model we found that the tidal gravimetric factor has an areal variation only 0.72 %.
According to Zharkov (1983) the uncertainty of the density function in the mantle ranges from 1 to 2 %. In view of this, we calculated *(Model B in Table 1)* the possible magnitude of areal variations and got 1.23 % (of course, while constructing Model B, we conserved the total mass and the total moment of inertia of the Earth).
It is interesting that Dehant and Ducarme (1987) got for the PREM practically the same result as we (1+h-3/2K=1.1564) But this and also our theoretical result differs from the mean of all observations (Melchior, 1983) (1+h-3/2K=1.161) by 0.5 %. And this deviation is very big if we remember that the possible range of variation of gravimetric factor is 0.7-1.2 %. In *Table 2* we estimate the formal error of gravity earth tide observations. The error value (0.3 %) is again too big if we are going to determine geographycal variations of gravity earth tide factor.

Table 2.Formal error of gravity Earth tide observations (Gerstenecker, Varga, 1985)

Error sources	*Amount of the error*
- Theoretical tides for the solid Earth	*< 0.1 %*
- R.M.S. error of observations	*0.2 %*
- Calibration error (on the basis of instrument comparison)	*0.5 %*
- Temperature and barometric influences (systematical part)	*0.1 %*
- Indirect effect of oceanic tides	*0.2 %*
The formal error value	*0.3 %*

It is easy to conclude from *Table 2* that the most important error source is the calibartion error . If we can reach calibration accuracy 0.1 the formal error of earth tidal observations became as big as 0.15 %. If we have this error value we are able to investigate the areal distribution of the gravimetric earth tidal factor.

B. STUDY OF THE PROBLEMS CONNECTED WITH THE NEWTONIAN LAW OF GRAVITATION

Recently many papers deal with the law of gravitation. At the same time a lot of problems are discussed in the literature in connection with the simple equation of Newton. The not solved problems in connection with the Newtonian law are:

- The gravity constant G is not known with needed accuracy. The inner accuracy of an individual gravity constant determination is much better as the agreement between the independent determinations carried out by different authors in diffe rent laboratories (*Table 3*). It can be concluded that the real accuracy of laboratory G values is 0.05 % and therefore we can say that the gravitional constant is the worst determined constant of nature.

Table 3 Laboratory G measurements

	$Gx10^{11}$
Facy, Pontikis (1972)	*6.6714+0.009 %*
Szagitov, et al.(1981)	*6.6744+0.012 %*
Luther, Towler (1982)	*6.6726+0.008 %*
The formal mean of laboratory measurements	*6.673 +0.045 %*

- the time dependence of the gravity constant. This possibility is discussed by many authors since the publication of Dirac's work in 1937. Different cosmological models, the theory of the expanding Earth is more or less connected with a hypothesis of a decreasing constant of gravity.
- the dependence of G on the composition of acting masses. It is expected on theoretical grounds (Fischbach et al., 1986; Schwazschild, 1986 etc.) and it is recently subject for many experimental researches.
- the possible difference between Newton's law valid for macroscopic and for laboratory (or small scale) ranges. The problem can be investigated with a comparison of Cavendish type laboratory G determinations with geophysical ones. The first atlempt of gravity constant measurements on scale much larger than the scale of laboratory determinations was done by G. B. Airy in the 1850 s. Similar work was done later on by Sterneck in Pribram in 1883. The principle of these so called geophysical G determinations is simple: a measurements of the gravity at the surface of the Earth is compared with the gravity determined under the surface. In this way we can eliminate from the calculations the mass of our planet and determine G separately. A collection of recent geophysical gravity constant determination are shown in *Table 4* on the basis of work Stacey et al. (1987).

Table 4 Geophysical G determinations (Stacey et al., 1987)

Author	*Depth (m)*	*Gx10*
Mc'Culloh (mine)	*0 - 648.8*	*6.733 + 0.060 %*
(1965)	*57.3 - 648.8*	*6.739 + 0.045 %*
	57.3 - 208.5	*6.724 + 0.119 %*
	223.0 - 389.0	*6.726 + 0.178 %*
	418.0 - 648.8	*6.746 + 0.193 %*
Hinze et al. (borehole, 1978)	*3712.0 -3963.0*	*6.810 + 1.028 %*
Hussian et al. (1981)	*251.0 - 590.0*	*6.705 + 0.239 %*
The formal mean of geophysical values		*6.740 + 0.189 %*

It can be concluded on the basis of a comparison of data listed in *Table 3* and *4* that the small scale determined G values are systematcally smaller as the geophysical ones. The difference is 1 %.
The weak point of G values obtained from surface and underground gravity measurements is the lack of detailed knowledge of density values between the surface and the level of the underground gravity measurements.
It seems to us that this last problem which is naturally of great importance can be studied on different background too. This can be for example a comparison of different gravity influences on gravimeters in other words we can study the problem calibrating gravimeters on different ways.

C. ABSOLUTE AND RELATIVE CALIBRATION OF THE GRAVIMETERS

In our former paper (Barta, Hajósy, Varga, 1988) we described how to calibrate (in absolute scale) with the use of induced gravity variations produced by a vertically moved heavy homogeneous circular ring. The positive features of such a gravity scale determination are:

1. the homogenity of the field at the extremums of the generated gravity effect
2. the raised and lowered around the instrument ring not loading the ground around the meter

3. the gravimeter remains stationary during the procedure
4. owing to technical reasons the gravity change brought by the ring is greater than that caused by other geometrically regular bodies (for example by a sphere).

We found earlier that in this way we can calibrate with absolute accuracy 0.1 % what is convinient for the solving problems connected both with the earth tides and the problem of different G values valid in macro- and microscopic ranges.

In both cases howerer we need two additional tools:

1. <u>relative calibration device</u> for the continuous monitoring the stability of the instrumental output;
2. <u>absolute calibration device</u> fixed to the meter to avoid the problems connected with the use of the measuring screw of the instruments.

1.) <u>Relative calibrtion device</u> was installed in our Askania type recording gravimeter in 1987 by Prof. M. Bonatz Two parallel plates were installed at the opposite to the mass end of the gravimeter arm. Introducing a constant voltage (we are using (15.000+0.001)V) a constant displacement appears on the output of the instrument. The rms error of a single displacement is better as 0.1 %. For the inner accuracy of 100 displacements we got $\pm$0.02 % On the basis of calibrations carried out once pro day during last two years we couldn't detect any statistically determined wariation of the records scale.

Using this electrostatic calibration device we were able to investigate the linearity of the instrument. In principle every instrument has deffects in his optical system because the non linear scaling of the micrometer screw, because the dead points in every mechanical systems.

Introducing artifical displacements of the beam of the meter with the electrostatic calibration device we could detrmine the output signal's scale (in microgal/mm or microgal/mV units) for different micrometer positions. We found out that in case of earth tide recording Askania type gravimeters the magnitude of relative scale variations is (1-1.5)% at the rms error level 0.3 %. It is also possible that the nonlinearity of the micrometric system has a certain time variation. The studies in this direction at the moment not gave for us unambiguos answer concerning the measure of the temporal changes in the nonlinearity. Naturally there are a lot of other both external and internal sources influences the scale of the output signal. We can study them separately it is a complex and labour-consuming work.

2.) Absolute calibration device

Because above problems both for the gravity earth tide studies and for the investigations in connection with the law of gravity we need beside the electrostatic control of scale a device for absolute calibrations. This can be for example a specially designed tilt platform. In this case we calibrating our equipment against the gravity field of the whole Earth. Using small A angles to incline the gravimeters the instrumental response is linear:

$$g=g(1-\cos A) = g(A2/2!-A4/4!+.....)=gA2/2 \qquad (1)$$

Naturally A must be small. In case of tilt equipment we used the basis of the tilt was L=500 mm and one turn of the screw of the platform (T) gives 0.5 mm.
In our work we have combined $T1=1.0$ and $T2=0.3T$ revolutions.
Let us suppose: the mass of the equipment is not in zero position but there is a deviation from A and it is X. In this case tilting the equipment by $\pm T1$ and $\pm T2$ we shall have

$$\begin{aligned} \Delta g1&=1/2g \cdot (T1+X+I1)\\ \Delta g2&=1/2g \cdot (T1-X-I1)\\ \Delta g3&=1/2g \cdot (T2+X+I3)\\ \Delta g4&=1/2g \cdot (T2-X-I3) \end{aligned} \qquad (2)$$

In (2) $I1$ and $I2$ are the additional tilt of the penduluw when $T1$ and $T2$ are used to tilt the whole gravimeter. If Yi is are output signal a combination of equations (1) and (2) gives for the record scale

$$K = \frac{T1 - T2}{\Sigma Y_{3,4} - \Sigma Y_{1,2}} \qquad (3)$$

Using this approach we got for the record scale determined with an accuracy 0.03 % *(Table 5)*

Table 5

Date	*K(microgal/mm)*	*Kmean(microgal/mm)*
24.01.89	*2.3526*	
25.01.89	*2.3522*	
06.02.89	*2.3520*	*2.3522+0.0001(0.03 %)*
07.02.89	*2.3523*	
08.02.89	*2.3519*	
02.03.89	*2.3520*	
05.03.89	*2.3521*	

It seems for us that the tilting of the gravimeters is an effective way for calibrating the output scale. It can be carried out in short time and during the whole procedure we are not disturbing the equipment itself. This way allows to us to go forward in both principal problems described in Sections A and B.

To get howerer reliable results we have to satisfy the following conditions:

1.) Daily temperature variations in the laboratory must be smaller as 0.05 C
2.) The plate of the instrument must be parallel to the plate of the tilting device. This condition can be satisfied with the examination of levels installed on the platform and one the instrument
3.) The beam of the meter must be parallel to the tilt. This position can be found tilting the gravimeter in different azimuts.

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