Elastic Constants of Rocks and the Velocity of Seismic Waves.

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The vibration of the earth's crust has from time to time been a favourite subject of discussion among the elasticians, and the propagation of seismic disturbance is a problem, whose solution has long been hoped for, both from the theoretical and the empirical point of view. With improved instruments, seismologists have recently determined the velocity of propagation with tolerable accuracy, but very little is known of the elastic nature of the medium through which the vibration has travelled. The resources from which physicists and seismologists draw their theoretical inferences are so scanty, that among the numerous rocks which constitute the earth's crust, only a few of the most commonly occurring rocks have had their physical properties investigated. The questions of elasticity, having close bearing with the deformation of the earth's crust, have repeatedly been a subject of research by several distinguished elasticians as Lord Kelvin, Boussinesq, Cerruti, and Chree. But we are baffled in our attempt to apply the result of subtle analysis to the actual problem, from the lack of our experimental knowledge as regards the elastic nature of the diverse rocks, which compose the outer coating of our planet. The present experiments were undertaken with a view to fill these gaps, and to supply on the one hand the wants of physicists, whose aim is to apply dynamics to the study of the geological phenomena, and on the other to meet the needs of seismologists, engaged in solving the problems touching the propagation of seismic waves.

Preparation of the Specimen.—The present experiments deal principally with the determination of Young's modulus and the modulus of rigidity, made on specimens of rocks which were easily accessible.

The number of rocks examined amounted to about eighty different specimens collected from various localities. These rocks were first cut in the shape of a rectangular parallelelopiped, and afterwards carefully polished into prisms of nearly 1 cm. square cross section and 15 cm. length. It was at first proposed to experiment with much larger specimens, but it was generally found impossible to find a large homogeneous piece with no trace of cleavage ; in addition to this, the apparatus with which the elastic constants were to be measured would become cumbrously large, and require great solidity, increasing at the same time the difficulties of experiment.

Most of the specimens were apparently isotropic, but on close examination it was found that the isotropy was only superficial. Rocks as slates with distinct sedimentation planes were generally cut parallel and perpendicular to them ; where such planes of symmetry were not easily discernible, the specimen was conveniently cut into prisms.

The thickness of these prisms was measured by a contact micrometer reading by means of a vernier to $\frac{1}{100}$ mm. at three different places in the middle line of two opposite faces ; namely, one at the middle and two at one quarter distance from the ends. The mean density of the prism was measured by dividing the mass by the volume, which was calculated from the known length and thickness. The density of several prisms cut from the same sample did not generally agree, showing that the material was only roughly homogeneous.

Modulus of Elasticity.—Young's modulus was measured by flexure experiment. The specimen to be tested was placed on two steel wedges, which served as fulcrums. The edge of the wedge was slightly rounded in order to prevent cutting on applying heavy weights. The flexure due to the weight hung at the middle of the prism was measured by means of a scale and telescope. By a special arrangement, a plane mirror was attached to the prism at the place where it rested on the wedges. The mirror was nearly vertical and the image of the vertical scale divided in mm., and placed at a distance of 2.73 m., was observed by a telescope provided with a filar micrometer. By this means, the deflection of 1″ was easily measurable.

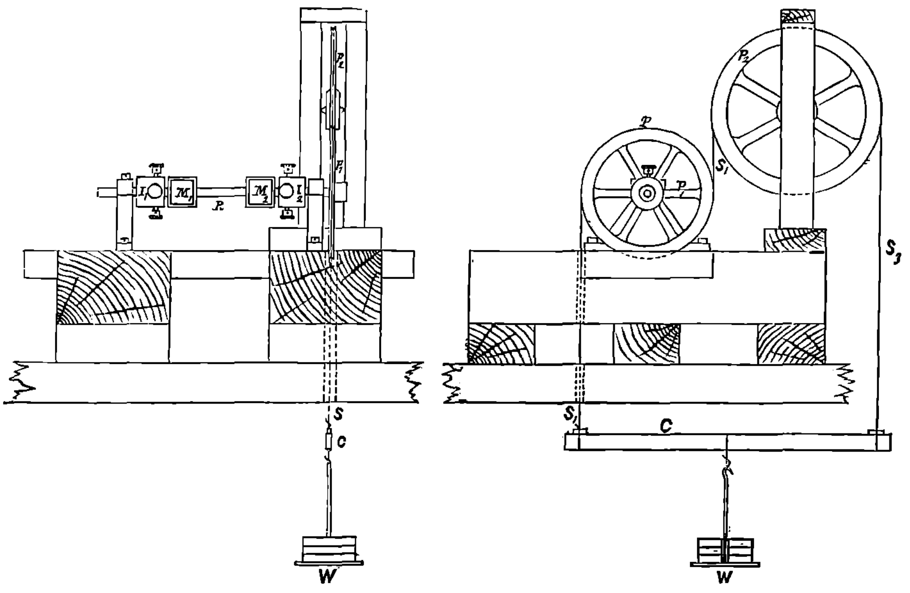
Denoting the length and the thickness of the prism by b and c resp., the distance between the fulcrums by a, and the angle of deflection by δ, we obtain for the modulus of elasticity E

$$E = \frac{3\,W\,a^2}{4\,b\,c^3\,\delta},$$

where W stands for the weight suspended in the middle of the prism.

The elastic heterogeneity of rocks called for the necessity of examining the constants in different directions; for this purpose, the prism was placed on its different faces on the fulcrum and the moduli for two mutually perpendicular directions were generally measured. These are denoted by E_1 and E_2 in the table of the elastic constants, and the mean of these two by E.

Modulus of Rigidity.—The modulus of rigidity was determined by measuring the amount of torsion produced by a given couple. It would lead too far if I attempt to describe the details of the instrument. The rectangular prism R was placed horizontal and firmly clamped at its both extremities to two solid pieces I_1, I_2 of iron. In order to prevent cracking by too firmly clamping, four small pieces of brass plates with thin sheet lead underneath was interposed between the four faces of the prism and the clamping screws. I_1 was fixed to a solid iron frame. The central steel cylinder protruding from I_2 was filed down to a sharp knife edge on its axis, coinciding with the central line of the prism. An agate plane attached to another solid iron frame supported the knife edge and the twisting pulley P. To the cylinder above referred to, a pulley P_1 of 14 cm. diameter was firmly fixed; a flexible string s_1 attached to a pin p on the circumference of the pulley passed over it, and was tied to a light wooden cross bar c. Another string s_2 was attached to the pulley, and instead of passing over it, was slung around another pulley P_2 such that the line of passage s_2 from P_1 to P_2 was vertical. The string on going over P_2 in



the opposite direction as the former string was again let down vertical and attached to the cross bar. By hanging the weight at the middle of the bar, the tension was the same in both strings and gave rise to a couple = radius of the pulley × weight. By this arrangement, the knife edge did not support the load producing the twisting couple, that of the prism, clamp and pulley being the only weight acting. The amount of torsion was measured by observing the deflection of two mirrors M_1 and M_2, one attached to the prism near the fixed clamp I_1 and the other near I_2. The deflections as measured by a vertical scale and two telescopes were generally large compared with those in flexure experiment, so that no micrometric measurement was needed. The difference of the two scale readings gave the torsion between the two places where the mirrors were fixed by special clamp screws.

Denoting the sides of the prism by b and c, the torsion for unit length by τ, the twisting couple by N, and the rigidity by μ, we get by St. Venant's formula for the torsion of a rectangular prism the following expression for N

$$N=\mu\tau b^3c\left\{\frac{16}{3}-\frac{b}{c}\left(\frac{4}{\pi}\right)^5\sum\frac{1}{(2n-1)^5}\frac{e^{\left(\frac{2n-1}{2b}\right)\pi c}-e^{-\left(\frac{2n-1}{2b}\right)\pi c}}{e^{\left(\frac{2n-1}{2b}\right)\pi c}+e^{-\left(\frac{2n-1}{2b}\right)\pi c}}\right\}.$$

It may be a question whether it is justifiable to use St. Venant's formula in the present experiment, as the boundry condition are somewhat different from those considered by St. Venant in deducing the above result. As the length of the prism was large compared with its thickness, and as the twist τ was measured at points not very near the ends of the prism, the result by using the above formula will not be materially different from the actual value. When the rock is of stratified structure and shows great difference in its elastic behaviour the formula will require modification, but in studying the elasticity of rocks in its broad feature, the modulus of rigidity calculated in the above manner will not be far from the general mean. The calculation of the series involved in the above formula is somewhat tedious. Fortunately, St. Venant has calculated a table of

$$\sum \frac{1}{(2n-1)^5} \frac{e^{\left(\frac{2n-1}{2b}\right)\pi c} - e^{-\left(\frac{2n-1}{2b}\right)\pi c}}{e^{\left(\frac{2n-1}{2b}\right)\pi c} + e^{-\left(\frac{2n-1}{2b}\right)\pi c}}$$

for different values of $\frac{c}{b}$. As the section of the prism was nearly square shaped, it was thought advisable to calculate the sum of the series at small intervals, when the ratio $\frac{c}{b}$ is nearly unity. As such tables will sometimes be found useful, I give the result of calculation in the following table.

Table of $\frac{16}{3} - \frac{b}{c}\left(\frac{4}{\pi}\right)^5 \sum \frac{1}{(2n-1)^5} \frac{e^{\left(\frac{2n-1}{2b}\right)\pi c} - e^{-\left(\frac{2n-1}{2b}\right)\pi c}}{e^{\left(\frac{2n-1}{2b}\right)\pi c} + e^{-\left(\frac{2n-1}{2b}\right)\pi c}} = \beta$

$\frac{c}{b}$	β	$\frac{c}{b}$	β
1.00	2.249	1.15	2.563
1.01	2.272	1.16	2.583
1.02	2.294	1.17	2.602
1.03	2.316	1.18	2.621
1.04	2.338	1.19	2.639
1.05	2.359	1.20	2.658
1.06	2.379	1.21	2.676
1.07	2.402	1.22	2.694
1.08	2.422	1.23	2.713
1.09	2.443	1.24	2.730
		1.25	2.748
1.10	2.464		
1.11	2.484		
1.12	2.504		
1.13	2.524		
1.14	2.543		

Hooke's Law and Elastic After-effect.—Preliminary experiments with granite showed that Hooke's law does not hold even for very small flexure and torsion, and that the after-effect is considerably great when the prism is sufficiently loaded or twisted; the deviation from the direct proportionality between the strain and stress was incomparably great compared with that observed in common metals. This will be chiefly due to the inferior limit of elasticity, so that it is necessary to experiment only within narrow limits of loading or twisting. These limits are widely different for different specimens of rocks, and the modulus of elasticity as well as that of rigidity was always determined with such stresses as will approximately produce the strain proportional to it.

The deviation from Hooke's law was prominent in certain specimens of sandstones, and it was the more marked in torsion than in flexure experiments. In certain rocks, it is indeed doubtful if anything like a proportionality between stress and strain can be found even for extremely small change of shape. On releasing these rocks from stress, the return to the former state is extremely small showing that the elasticity of rocks is of very inferior order. The elastic yielding of rocks under continuous action of stress is very remarkable as the following readings of the deflection in the experiment on torsion will show.

SPECIMEN: IZUMI SANDSTEIN.

$a = 100.0$ mm., $b = 10.12$ mm., $c = 10.09$ mm.,

Torsional loading : 400 grms

Zero reading before loading : 24.2

Loaded : 2^{h} $18.^{m}0$ Sept. 10, 1898

Time.		Reading.
h	m	
2	18.1	72.0
	18.5	75.3

Time.	Reading.
$2^{h}\ 19.0^{m}$	77.1
19.5	78.1
20.0	78.9
20.5	79.6
21.0	80.1
21.5	80.6
22.0	81.0
23.5	81.4
23.0	81.8
23.5	82.1
24.0	82.4
25.0	83.6
27.0	84.1
28.0	84.5
29.0	84.9
30.0	85.2
31.0	85.5
32.0	85.9
33.0	86.2
34.0	86.5
35.0	86.8

It will be seen that the initial deflection amounts to 47.8 mm.; the torsion of the prism gradually increases in course of a few minutes, so that after a lapse of about 19 minutes, the increase of deflection is nearly 30 per cent of the initial. The increase becomes asymptotic with time.

The above mentioned property of rocks will be of no small interest in dynamical geology as it naturally illustrates the possibility of the folding of rocks and other kindred phenomena pertaining to the manifold change of shape in rocks, wrought by the continuous action of stress.

Velocity of Elastic Waves.—It was my intention to determine the modulus of elasticity, and then calculate the velocity of propagation of the longitudinal as well as that of the transversal waves, on the supposition that the material is isotropic. Few experiments with rocks of different ages showed that these attempts are for the most part fruitless, as the assumption of isotropy was not generally admissible. With archæan and palaeozoic rocks, it was possible to sort them into proper shape for experiment only in a certain direction, as they were generally of schistose structure, and extremely brittle in the direction perpendicular to it ; in such cases the elastic behaviour was of course widely different in these directions. Even with granite which apparently is homogeneous in structure, the difference of elasticity with direction was noticed. On enquiry these rocks were pressed from one side during its formation, and thus left its trace in the relation of strain to the stress. For the complete discussion of the elastic nature of these rocks, the determination of the moduli of elasticity and of rigidity considered as an isotropic substance is insufficient ; we are in fact dealing with quasi-crystalline bodies, so that the number of elastic costants must depend on the number of symmetry planes, which can be drawn in these rocks. The type of the elastic waves travelling in such a medium will be determined, when all of these constants are known. As we have no simple means of examining these symmetry planes, a single modulus of elasticity and rigidity was determined, on the supposition that the material is isotropic.

In the discussion of the propagation of seismic waves, we have to deal with wave-length which measures over a kilometre. Geologists tell us that uniform strata of a kilometer thickness are of rare occurrence, and it may be doubted if these waves do not suffer change of type and shape in traversing the earth's crust. Unquestionably longitudinal plane waves whose velocity of propagation in an isotropic medium is given by the formula $\sqrt{\frac{\lambda+2\mu}{\rho}}$ (following Lamé's notation) would seldom come into existence. A complete discussion of waves in quasi-crystalline rocks requires complicated analysis, which necessitates the knowledge of the elastic behaviour of rocks cut in various directions. To obtain a general view of the propagation, I have thought it advisable to calculate $V_l=\sqrt{\frac{E}{\rho}}$ for the longitudinal waves. Suppose the Young's modulus E is determined by flexure experiments on a prism cut parallel to a plane of symmetry, then V_l will give the velocity of longitudinal wave travelling along the prism. The velocity in the sense above explained is given under V_l and the velocity of the transversal wave $\sqrt{\frac{\mu}{\rho}}$ under V_t. I do not mean to say that the actual velocity of longitudinal waves in various rocks is given by V_l but when such values are not obtainable, V_l will probably give a rough estimate. The elastic constants of rocks are tabulated in the order of geological age; for the same geological age, those with larger velocity of propagation V_l come before those with the slower.

ELASTIC CONSTANTS OF ROCKS.

Rock	Specimen No.	ρ	E_1 (C.G.S.)	E_2 (C.G.S.)	E (C.G.S.)	μ (C.G.S.)	$V_l \frac{kilm.}{sec.}$	$V_t \frac{kilm.}{sec.}$
ARCHAEAN ROCKS.								
Chlorite Schist (*Chichibu*)	9	2.977	112.1×10^{10}	132.4×10^{10}	122.3×10^{10}	24.03×10^{10}	6.40	2.84
,,	50	2.955	146.0	147.6	146.8	31.57	7.05	3.27
(Eruptive)								
Peridotite Serpentine (*Kuzi*)	16	2.825	72.92	58.99	65.96	22.24	4.83	2.81
Peridotite Serpentine	41a	2.777	62.42	55.86	59.14	20.09	4.61	2.69
	41b	2.786	54.15	53.90	54.03	19.73	4.41	2.66
Ophicalcite	45	2.593	38.90	53.71	46.31	......	4.22	
Peridotite Serpentine	17	2.570	39.03	46.00	32.52	16.00	4.07	2.49
PALAEOZOIC ROCKS.								
Schalstein (*Rikuchyū*)	79	2.653	120.50	92.25	106.4	18.90	6.32	2.67
Clayslate (*Nikkō*)	74	2.149	79.69	83.29	81.49	28.06	6.16	3.61
Schalstein (*Rikuchyū*)	78a	2.768	70.02	95.00	82.51	25.36	5.45	3.03
	78b	2.772	97.90	103.30	100.60	21.25	6.02	2.77
Sandy Slate (*Rikuchyū*)	73	2.640	81.79	92.40	82.10	17.05	5.75	2.54
Clay slate	2a	2.674	98.00	83.09	90.55	13.79	5.82	2.27
	2b	2.690	90.64	86.71	88.68	20.75	5.74	2.78
	2c	2.708	51.92	62.26	57.09	20.74	4.52	2.77
Limestone (*Musashi*)	55	2.630	84.95	88.45	86.20	29.83	5.74	3.38
Limestone	13	2.653	80.20	86.61	83.40	31.00	5.60	3.42
Limestone (*Musashi*)	29	2.682	68.86	79.55	74.20	21.71	5.26	2.84
Weathered Clayslate	1a	2.314	62.15	61.35	61.75	10.03	5.18	2.08
	1b	2.304	56.83	58.90	57.87	8.85	5.01	1.96

Rock	Specimen No.	ρ	E_1 (C.G.S.)	E_2 (C.G.S.)	E (C.G.S.)	μ (C.G.S.)	V_l $\frac{\text{kilm.}}{\text{sec.}}$	V_t $\frac{\text{kilm}}{\text{sec.}}$
Marble	11a	2.654	76.0×10^{10}	63.72×10^{10}	69.86×10^{10}	30.11×10^{10}	5.13	3.37
	11b	2.625	63.53	46.2	54.86	28.60	4.54	3.45
Schalstein	80	2.824	74.60	70.52	72.56	18.96	5.07	2.58
Schalstein (*Tosa*)	75	2.762	57.68	37.70	47.69	8.98	4.63	1.80
Weathered Clay slate	60a	2.316	39.44	35.27	37.36	4.99	4.02	1.47
	60b	2.306	35.37	36.69	36.03	5.27	3 96	1.51
Marble	12a	2.650	37.26	37.64	37.45	15.08	3.76	2.39
	12b	2.650	37.33	28.33	32.82	18.80	3.93	2.66
Clayslate (*Tanba*)	3a	2.384	34.48	30.76	32.62	8.00	3.70	1.83
	3b	2.392	30.64	30.35	30.50	8.54	3.57	1.87
Contact Clayslate (*Mikawa*)	64a	2.462	30.35	28.10	29.23		3.45	1.71
	64b	2.416	31.00	31.86	31.43	......	3.61	......
Weathered Clayslate	7a	2.503	12.45	12.20	12.33	4 60	2.32	1.36
	7b	2.500	13.00	13.64	13.32	4.31	2.31	1.31
Weathered Clayslate	65a	2.490	12.72	12.26	12.49	6.59	2.24	1.63
	65b	2.500	12.54	12.47	12.51	4.43	2.24	1.33
(Eruptive)								
Granite (*Shōdoshima*)	69	2.572	37.91	46.71	42.31	18.43	4 05	2.68
Granite	42	2.550	31.42	......	......	13.99	3.51	2.34
Granite (*Hitachi*)	68	2.549	18.83	20.43	19.63	6 89	2.78	1.64
Granite (*Hitachi*)	71	2.590	14.84	15.12	14.98	5.05	2.42	1.40
Granite	52	2.503	15.23	9.73	22.48	5.47	2.22	1.48
Granite (*Hitachi*)	56	2.530	11.97	9.89	10.93	4 43	2.08	1.32

MESOZOIC ROCKS.

Izumi Sandstein	5	2.216	9.2	9.0	9.12	3.1	2.03	1.18
	6a	2.236	7.1	7.2	7.12	2.4	1.78	1.04
	6b	2.223	7.7	7.6	7.67	2.7	1.86	1.10

Rock	Specimen No.	ρ	E_1 (C.G.S.)	E_2 (C.G.S.)	E (C.G.S.)	μ (C.G.S.)	$V_e \frac{\text{kilm.}}{\text{sec.}}$	$V_t \frac{\text{kilm.}}{\text{sec.}}$
Schalstein	77	2.778	75.7×10^{10}	83.0×10^{10}	79.4×10^{10}	23.2×10^{10}	5.35	2.89
Clayslate (*Rikuchyū*)	72	2.711	88.4	99.3	98.8	22.6	5.88	2.89
Clayslate (*Rikuchyū*)	53	2.702	83.6	85.3	84.5	18.5	5.59	3.17
Clayslate (*Tsushima*)	62a	2.681	32.2	50.6	41.4	14.8	3.91	2.35
	62b	2.678	43.7	44.3	44.0	14.2	4.06	2.31
CAINOZOIC ROCKS (Tertiary)								
Rhyolite (*Izu*)	51	2.316	32.1	17.5	24.8	14.0	3.24	2.46
Rhyolte Tuff (*Iyo*)	8a	2.346	21.9	21.5	21.73	9.32	3.05	1.99
	8b	2.316	21.8	20.0	20.90	8.05	3.01	1.86
Tuff Sandstone (*Kōzuke*)	19a	2.305	20.6	21.1	20.8	8.74	3.02	1.95
	19b	2.321	21.2	21.4	21.3	8.45	3.02	1.91
Rhyolite (*Kōzuke*)	59a	2.472	21.3	18.7	20.0	8.57	5.85	1.86
	59b	2.454	19.5	18.3	18.9	9.15	2.78	1.93
Rhyolite Tuff (*Mikawa*)	63a	2.228	18.8	19.9	19.3	6.9	3.00	1.79
	63b	2.198	17.4	11.8	14.6	......	2.59	
Rhyolite (*Izu*)	27a	1.945	11.3	11.7	11.5	5.78	2.43	1.72
	27b	1.944	14.0	15.1	14.6	5.86	2.74	1.74
Rhyolite Tuff	32	1.889	8.1	10.1	9.1	4.2	2.20	1.49
Sandstone (*Chōshi*)	58	2.345	10.9	11.4	11.2	4.60	2.18	1.40
Rhyolite Tuff (*Amakusa*)	66	2.263	8.00	7.59	7.80	3.59	1.86	1.26
Rhyolite Tuff (*Iwashiro*)	61a	2.228	10.8	11.1	10.96	6.25	2.22	1.51
	61b	2.198	9.8	9.6	9.67	5.66	2.10	1.67
Rhyolite Tuff (*Tochigi*)	43	1.371	1.43	2.49	1.96	1.06	1.19	0.89
(Diluvium)								
Tuff	36	1.850	35.7		......	6.235	4.39	1.84
Andesite	54	2.557	43.9	45.8	44.9	18.50	4.19	2.69
Andesite	70	2.462	45.5	26.7	36.1	11.69	3.80	2.18
Tuff	30	2.169	28.3	27.6	27.95	10.99	3.59	2.25

Rock	specimen No.	ρ	E_1 (C.G.S.)	E_2 (C.G.S.)	E (C.G.S.)	μ (C.G.S.)	$V_e \frac{\text{kilm.}}{\text{sec.}}$	$V_t \frac{\text{kilm.}}{\text{sec.}}$
Andesite	15	2.201	29.2	23.6	26.38	12.57	3.45	2.39
Tuff	10	2.283	24.3	24.9	24.62	10.74	3.28	2.17
Tuff	14	2.222	21.6	22.8	22.2	8.48	3.18	1.96
Andesite	28	2.165	19.46	27.75	23.51	12.15	3.24	2.37
Andesite	39	2.397	23.07	20.4	21.73	10.13	3.01	2 06
Tuff	20	1.859	14.4	14.5	14.41	5.07	2.79	1.65
Tuff	4a	1.838	10.9	11.86	11.40	4.56	2.99	1.58
	4b	1.817	12.0	12.60	12.33	3.88	2.60	1.46
Andesite	40	2.302	14.76	12.6	13.68	5.99	2.44	1.61
Tuff	57	2.039	11.26	10.70	10.98	5.51	2.32	1.65
Andesite Tuff (*Echizen*)	67a	2.435	13.15	12.77	12.96	5.78	2.31	1.54
	67b	2.400	13.57	13.21	13.39	5.55	2.37	1.52
Andesite	38	1.943	10.30	10.39	10.35	4.13	2.31	1.46
Andesite	49	2.158	8.96	13.1	21.0	5.26	2.26	1.56
Andesite Tuff	23	1.829	8.23	8.48	8.36	3.92	2.14	1.46
Andesite	34	2.022	9.17	8.44	8.81	6.00	2.09	1.72
Andesite	47	2.425	8.51	8.38	8.45	4.06	1.86	1.29
Tuff (*Izu*)	31	1.915	7.53	5.82	6.68		1.86	......
Tuff	33	1.819	6.23	6.42	6 33		1.87	......
Andesite	46	2.574	8.87	8.36	8.62	2.92	1.83	1.07
Andesite (*Izu*)	25a	1.984	6.57	5.12	5.85	1.236	1.72	0.79
	25b	1.632	5.57	5.14	5.36	1.63	1.60	0.88
Andesite	48	2.102	5.51	6.81	6.16	2.47	1.71	1.08
Andesite Tuff	21	1.497	3.74	4.12	3.93	1.39	1.62	0.97
Tuff (*Izu*)	35	1.286	3.45	3.31	3.38	1.50	1.62	1.08
Tuff (*Awa*)	44	1.448	2.72	3.87	3.30	1.17	1.50	0.90
Quartz Sandstone	24	2.138	4.04	4.05	4.05	1.30	1.37	0.78
Quartz Sandstone	37	2.230	4.02	......	4.02	......	1.34	......

Some of the specimens which have been examined are nearly isotropic. Most of these rocks are of recent formation. For these, I have calculated the velocities of propagation of longitudinal waves in unlimited medium $V = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ $\left(= \sqrt{\frac{k + \frac{4}{3}n}{\rho}} \text{ using Lord Kelvin's notation} \right)$, which are placed under the following table.

Rock	Age	Density	$V = \sqrt{\frac{\lambda + 2\mu}{\rho}} \left(\frac{\text{kilm.}}{\text{sec.}}\right)$
Peridotite Serpentine	Algonkian	2.786	5.86
Marble	Palaeozoic	2.654	4.09
Weathered clayslate	„	2.490	2.25
Idzumi sandstein	Mesozoic	2.236	2.93
„	„	2.223	2.76
Tuff sandstone	Tertiary	2.321	3.35
„	„	2.305	3.16
RhyoliteTuff	„	2.316	3·18
„	„	2.346	3.11
Rhyolite	„	1.944	3.02
„	„	2.454	2.78
Rhyolite Tuff	„	2.228	2.25
„	„	2.198	2.14
„	„	2.263	1.88
Tuff	Diluvium	2.557	4.[illegible]4
„	„	2.167	4.02
„	„	2.222	3.77
„	„	2.283	3.38
Andesite	„	2.397	3.06
Tuff	„	1.838	2.75
Andesite Tuff	„	2.014	2.58
Andesite	„	2.547	2.57
„	„	1.943	2.54
Andesite Tuff	„	2.400	2.50
„	„	2.435	2.35
Tuff	„	2.039	2.32
Andesite	„	2.022	2.21

I did not think it necessary to calculate the velocity of surface waves, which according to Lord Rayleigh amounts to $0.9554 \sqrt{\frac{\mu}{\rho}}$, as the difference of rigidity in different specimens is so great that the presence of the factor 0.9554 will not materially affect the result.

General Result.—In examining the elastic constants of rocks classified according to the age of formation, we find a distinguished gradation as we pass from those of recent formation to the oldest. The increase of density as well as the quasi-crystalline behaviour of rocks are the most important characteristic of rocks, which are deeply embedded in the earth's crust. The chlorite schist of Chichibu has a density nearly equal to 3, although its modulus of elasticity is greater than that of brass or copper with a rod cut in the direction of strongest tenacity, it is so brittle in the direction perpendicular to it that it is impossible to obtain a single specimen with which the elastic constant can be accurately determined. The elastic constants are widely different as the specimen is cut in one or other direction especially in archaean and palaeozoic rocks, as schists and slates with distinct sedimentation planes. Rocks of eruptive origin are generally free from such directional behaviour, but when they are pressed or otherwise subject to continuous application of stress, the difference of elasticity in different directions can still be traced. Such appears to be the case with marble and granite.

The elastic constants of archaean and palaeozoic rocks are far superior to those of the cainozoic, but the velocity of propagation of longitudinal or transversal waves is not proportionally large. As the ratio of the elastic constant to density determines the velocity of propagation, we can not at once conclude from the increase of elasticity that the waves travel with greater velocity. It would be too bold to draw anything like a general conclusion from the examination of some eighty specimens, but so far as the present experiments go, the tendency is such that the elastic constants increase more rapidly than the density as the rock becomes denser ,and consequently elastic waves travel with greater velocity in the interior than on the surface of the earth's crust. Eruptive rocks are more isotropic than those of nonigneous origin, and have inferior elasticity, but there is the same distinction with age. Elastic waves in eruptive palaeozoic rocks travel with slower velocity than in those of the archaean of the same origin; a similar remark applies to cainozoic rocks with a few exceptions.

As we go deep in the earth's crust the rocks generally assume schistose structure, we have reason to believe that the elastic constants of the constituent rocks increases in a certain particular direction, which evidently conicides with that of swiftest propagation of elastic disturbance. Pressed by the weight of the superincumbent crust these rocks will be of greater density, so that the increase of elastic constants is attended with corresponding increase of density. We can not conceive that the elastic constant nor the density will continually increase as we approach the centre of the earth; they will both attain asymptotic values. The alternatives are either the ratio of elastic constants to density goes on gradually increasing, or it first reaches a maximum and then goes on decreasing. The former supposition makes the velocity of elastic waves increase from the surface towards the centre of the earth, while the latter implies the existence of *the stratum of maximum velocity of propagation.* Such a stratum, if it exists, will lie pretty deep in the earth's crust and will be inaccessible to us, but the question will be settled by the seismologists.

Velocity of Propagation of Seismic Waves.—A glance at the table of elastic constants will show the complex elastic nature of rocks composing the earth's crust. The path pursued by waves of disturbance must necessarily assume very complicated forms, as they are subject to manifold reflection, refraction, and dispersion. We can perhaps borrow analogy from a kindred optical phenomenon of curved rays in a medium of heterogeneous density, studied experimentally by Macé de Lépinay and Perot, and theoretically discussed by A. Schmidt and

Wiener. The phenomena presented by the seismic wave will be of still more complex character as the medium is of quasi-crystalline nature, and the wave may suffer refraction something akin to that of light in iceland spar and arragonite. The elastic constants of rocks through which the disturbance propagates will rarely satisfy the condition of giving rise to purely longitudinal or distortional waves, so that the seismic wave will be of a mixed character. What Mr. Milne designates earthquake echos or reverberations will partly find explanation in the intricate behaviour of diverse rocks against the elastic wave travelling through them. The waves propagating from the centre of disturbance will appear on the seismograph as undulations of irregular periods, especially near the origin. At a distance, waves of short period will gradually die out owing to the greater damping effect, while those of long period will still leave their mark, although not felt by us as a shock.

The investigation of the seismic waves affords the best means of feeling the pulse of the interior of the earth ; the elastic nature and the density distribution of the constituent rocks, or even the condition of the inaccessible depth will in some future day be brought to light by the patient study of the disturbance, which traverses the strata of heterogeneous structure and appears as tremors or earthquakes on the earth's surface. I think the introduction of the horizontal pendulum is a great progress in that branch of study, which relates to the earth's interior, not that it records the apparent surface movement of the soil, but that it does not fail to record earthquakes of distant origin, which though insensible to us, sometimes appear as slow waves of gigantic amplitude. By it will be found disturbances, which came through various strata, and probably those travelling through the stratum of maximum velocity of propagation.

Seismic waves travelling through strata of heterogeneous elasticity and density will generally be not purely longitudinal as in the case of sound, nor purely transversal as in the case of light, but a mixture of these two kinds. The velocity of propagation expressed as functions of elastic constants and density is not a simple problem and moreover we do not possess sufficient experimental data to test the result of calculation. The formula $V_1=\sqrt{\frac{E}{\rho}}$ for longitudinal waves in a thin rod will give a rough estimate of the velocity.

From records taken in Italy and Japan, Professor Omori concludes that the velocity of the first tremor is almost always equal to 13 kilometers per sec. The question naturally arises : how can we account for such enormous rate of propagation ? The velocity of plane longitudinal waves in an infinite medium of steel is about 6.2 kil. per sec. ; if we take a rod of steel in place of an uniform medium and give a blow to one of its ends, the longitudinal wave will travel with a velocity of 5.3 kilometres ; if the same experiment be repeated on a piece of iron pyrites cut parallel to its axis of greatest elasticity, the velocity will be 8.4 kil. per second ; in topaz, it will amount to 9 kil. Thus even with substances easily accessible on the earth's surface, we have instances of elastic waves travelling with a velocity of something like 10 kil. In the present experiments the velocity in several primeval rocks ranges from 6 to 7 kil. per sec.; as we go deeper in the crust, we may not fail to find those rocks, whose elastic constants are several times greater than those near the surface. So far as I am aware iron pyrites has the greatest modulus of elasticity among the substances, which have till now been placed under experimental test ; it is about 1.6 times greater than in steel and amounts to 3.5×10^{12} C.G.S units (Voigt). If we now imagine a stratum in which Young's modulus exceeds that of iron pyrites as much as that of iron pyrites exceeds that of steel, we can realize a velocity ascribed by seismologist, had not the increase of density been so great as to bring down the rate of propagation. The velocity of 13 kilm. per second, which is that calculated from the preliminary tremors, will roughly correspond to $E=6.0\times10^{12}$ and $\rho=3.5$. To speak of the relation between density and elastic constant might seem a little absurd, but in the rocks so far examined, certain relation between these two physical constants seems to exist. Comparing the elastic constants of cainozoic and archaean rocks, we find that with the increase of density from 2 to 3, the modulus of elasticity has increased more than ten times in certain specimens. Thus it would not be a wild conjecture to put $E=6\times10^{12}$ when the density is 3.5. As the mean density of the earth is little over 5.5, we shall come across a stratum of the density above cited not very far from the surface. These considerations give support to the view above stated that there is a stratum of maximum velocity of propagation.

Elastic waves travel with slow velocity in surface rocks. If the principal shocks in the seismometer record be taken into account, the velocity turns out to be very small and about 3.3 kilm. This evidently is about the mean velocity of propagation in most of the surface rocks, and shows that waves of large amplitude creep along the surface. It is not wonderful that with distant earthquakes, the duration sometimes extends over several hours, as the disturbance travels through strata of different elastic constants and the wave modified in various ways will appear all blended together on the seismograph. Although 3 kilm. may be a mean velocity, there are certain surface rocks in which the velocity is less than a kilometer. The shock at the epicentre may last only for a short time, but the duration at a distance will be lengthened, as the range of velocity is very wide. The disturbance coming from the strata of greatest rate of propagation will first make its appearance as the beginning of the preliminary tremor, followed by waves travelling with slower velocity till the principal shock arrives as surface waves. It will be followed by waves travelling with still slower velocity leaving faint record on the seismograph, till they at length fade away. Neglecting the time of passage from the stratum above mentioned to the surface, it is natural to expect that the duration of the so-called preliminary tremor preceding the earthquake shock increases *linearly* with the distance of the epicentre from the place of observation. The above relation was established from various earthquakes which happened in Japan, recorded by Prof. Omori.

With great earthquakes which are perceptible on a seismograph at very great distances, the duration will continually increase with distance; the disturbance may sometimes propagate still unabated in one or other direction round the earth. If the last mentioned case actually take place, the tremor will probably last even for days. As such records have sometimes been obtained by seismologists, it may not be out of place here to notice the possibility for such undulatory movement of the ground.

In conclusion, I wish to express my thanks to Professor Koto and Mr. Fukuchi for valuable information concerning the geological and petrological character of rocks examined in the present experiment.