Terzaghi and Liquefaction Flow Slides

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Abstract: Terzaghi had a clear understanding in "Erdbaumechanik" of the mechanics of liquefaction flow slides. This paper summarizes a number of recent developments that contribute to our understanding of the triggering of "spontaneous liquefaction" associated with the initiation of such flow slides.

1. Introduction

It is most fitting to celebrate the 75th anniversary of the publication of Terzaghi's "Erdbaumechanik" (Terzaghi, 1925). As emphazied recently (Morgenstern, 2000) this publication was a pivotal event and marked the emergence of modern geotechnical engineering with its many achievements and on-going challenges.

As one thumbs through this seminal volume, it is not surprising to find comprehensive discussions on the description and classification of soils with some emphasis on the Atterberg Limits, as well as discussions on the nature of friction in soils and the role of pore water pressure within the context of changes in effective stress and associated consolidation. Earth pressures and many aspects of foundation engineering are also treated in depth.

The problem of liquefaction flow slides, particularly in the marine environment, has always been of interest to the Author. It is perhaps not so well recognized that Terzaghi also discussed this phenomenon with remarkable insight (p. 347). He had a clear understanding of the role of water pressure in creating flowing sand (Schwimmsanderscheinungen) and utilized it to explain the flow slides that had been described along the coast of Zeeland by Müller (1898). Even in 1925, Terzaghi characterized the transformation into a flow slide as spontaneous and later he used the term "spontaneous liquefaction". The purpose of this paper is to summarize the advances made in understanding the mechanics of spontaneous liquefaction, building on the initial framework put forward by Terzaghi.

The collapse of wet granular materials can have catastrophic consequences with respect to loss of life and property damage. The high mobility of these materials in the liquefied state has a bearing on many problems of practical interest such as stability of waste dumps, debris flows, mechanics of submarine landslides, stability of artificial islands, stability of hydraulic fill dams, to identify a few. That such instability can be triggered by dynamic effects is well-recognized, but it is not so well known that static triggers also exist.

2. Liquefaction flow slides

Liquefaction flow slides are among the most treacherous of mass movements. Hutchinson (1988) has defined a liquefaction flow slide as:

"characterized by the sudden collapse and extensive, very to extremely rapid run-out of a mass of granular material or debris, following some disturbance. An essential feature is that the material involved has a meta-stable, loose or high porosity structure. As a result of the disturbance this collapses transferring the overburden load wholly or partly onto the pore fluid, in which excess pressures are generated.

The consequent loss of strength gives the failing material, briefly, a semi-fluid character and allows a flow slide to occur".

Liquefaction flow slides occur in both natural and man-made deposits. They are particularly catastrophic geotechnical phenomena because they occur suddenly, with little or no warning, and their mobility can threaten distant people and property. The sudden onset of a flow slide precludes reliance in geotechnical practice on many aspects of the observational method, which is otherwise so powerful.

The reduction to almost zero effective stress by induced pore pressures is commonly termed liquefaction. The elements of liquefaction have been understood for a long time as has been the relationship between flow slide mechanics and liquefaction.

However the initiation of flow slides has not been well understood and recent developments have contributed to a better understanding of them. By improving our understanding of the mechanics of flow slides, we will be better-equipped to evaluate such hazards and to eliminate them if necessary.

3. Significant examplex of flow slides

In the classical paper by Casagrande (1936), the earlier studies of Terzaghi are cited and additional examples of flow slides due to the failure of a hydraulic fill dam in Russia and the collapse of a chemical waste deposit are described. Casagrande notes perceptively:

"A serious disturbance, for example, an earthquake, or possible even large seepage pressures, may result in a tendency to reduce the volume due to an adjustment in the structure, reducing the shearing resistance to such a point that the whole mass can flow like a liquid under the effect of the seepage pressures".

That is, in the earliest studies it was recognized that liquefaction flow slides could be induced by both dynamic and static triggers. It was also recognized that a mass of soil will not necessarily be stable if the slope is less than the angle of repose. As will be noted later, this lesson has been lost in some industrial practice.

Casagrande (op. cit.) reported an elegant experiment to demonstrate the triggering of liquefaction. A tank was filled with a fine quartz sand in a loose, saturated state, with free water standing on its surface, and a weight was placed on the surface. Then a stick was thrust into the sand, some distance from the weight, and the weight suddenly sank. The slight but rapid deformation produced by the penetration of the stick created local pore pressures which then propagated progressively throughout the entire soil mass resulting in total loss of bearing capacity.

There is a number of instances in the literature of hydraulic fill dams collapsing and transforming into liquefaction flow slides. As a result, hydraulic fill methods of construction fell out of favour for civil engineering works in North America and Western Europe in the 1930's. One should note that while the collapse of the Fort Peck Dam in 1938 constitutes a watershed in the evolution of hydraulic fill methods for dam construction in North America and marks its decline, the most comprehensive investigations concluded that the fault lay with the weak foundation and that partial liquefaction of the material in the slide may have only affected the extent to which the slide progressed upstream (Middlebrooks, 1942). The evolution and application of hydraulic fill methods of construction has been summarized by Morgenstern and Küpper (1988).

Liquefaction flow slides are often characteristic of submarine slope failures and Terzaghi (1956) remains an important reference in this regard. He observed that slopes composed of coarse-grained sediments (coarse sand or gravel) do not fail unless the slope angle is equal to the angle of repose, and even then, the slide has the character of a minor, local slump. The general slope conditions remain unaltered. After aggradation has stopped, the slope remains stable unless it is undercut. However slopes of finer-grained cohesionless or almost cohesionless sediments have failed, even though the slope angle was very much smaller than the angle of repose of the material and aggradation had stopped long ago. He described several cases of submarine slope failures where only minor triggering mechanisms could be detected and referred to this phenomenon as spontaneous liquefaction. A review of case histories suggested that liquefaction flow slides could be induced by earthquakes, blasting operations, seepage pressures exerted by the flow of the ground water towards the ocean during very low tides and other agents. Details remained unquantified and enigmatic.

Subsequent studies into the mechanics of submarine landslides by Morgenstern (1967) indicated that the initiation of liquefaction flow slides was consistent with the mobilization of the undrained strength of loose cohesion less sediments and it was implicit that an undrained trigger such as an earthquake or wave effects was essential to mobilize the undrained strength.

Flow sliding in the Fraser River delta has been a recurrent phenomenon. Five major slides have occurred in the delta between 1970 and 1985. McKenna et al. (1992) have summarized the geological setting and described the circumstances. A more recent flow slide occurred in December, 1992, with an approximately 23° slope, about 10 m high failing and the silty sand flowing out to deeper water down a submarine channel. However, in this instance instruments had been put in place by the University of Alberta to monitor any wave-induced or other triggers prior to collapse. None were detected. From these field studies alone, the mechanics of triggering remained unexplained.

The influence of earthquake loading on the build-up of pore pressure and subsequent liquefaction of cohesionless soils has received a considerable amount of attention in the literature. The most common manifestation of liquefaction flow sliding are the lateral spreads that have developed after many earthquakes. However the best documented case history is the collapse of the upstream face of the Lower San Fernando Dam in 1971 (Seed et al., 1988). It is important to note that the flow slide started after the earthquake motion had ended. That is, the influence of the seismically induced cyclic stresses served only to trigger the flow slide. The movements were driven by gravity stresses as opposed to dynamic forces.

The collapse of the Stava tailings dam in 1985 resulted in 268 deaths and considerable property damage. The circumstances surrounding this occurrence have been described by Berti et al. (1988). Figures 1 and 2 illustrate a section and schematic layout of the waste storage scheme. Several explanations for the failure were advanced in the cited reference, but the whole story was not covered.

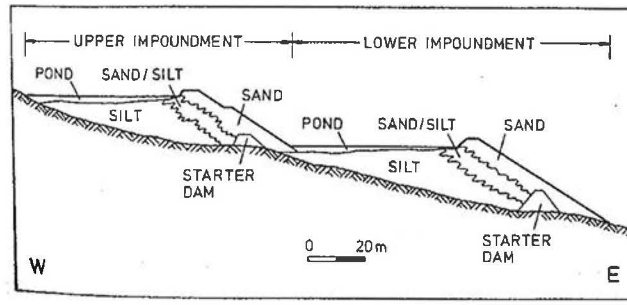

Fig. 1: Schematic section of Prestavel tailings dams (Berti et al., 1988).

The Author and Dr. E. D'Appolonia had been retained by one of the defendants in the litigation. It was their view that the dam operators at the time of the failure were constructing the dam improperly. They were not controlling the beach effectively during upstream construction and water was allowed to enter the steep loose face of the upper dam. This induced static liquefaction of the face, which then allowed the weak slimes to fail in an undrained manner. Additional details are provided by Morgenstern (1996).

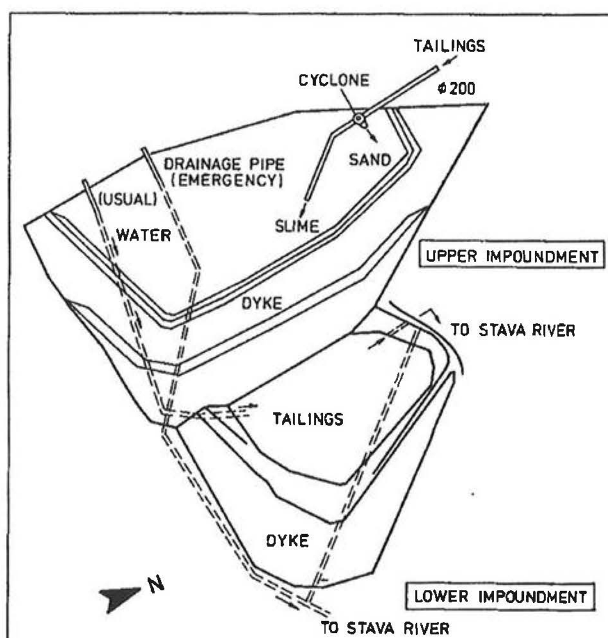

Fig. 2: Schematic layout of Prestavel tailings dams (Berti et al., 1988).

Other occurrences of static liquefaction arose during the construction of the sub-sea berm at Nerlerk, intended to be used for offshore oil exploration in the Beaufort Sea (Sladen et al., 1985 (a)). Several slides developed, triggered by loading associated with the hydraulic placement of sand. The void ratio of the sand within the berm was approximately 0.8 with a permeability in the order of 10^{-5} m/s. The initial slopes and height of the berm were in the range between 10° to 12° and 25 to 30 m respectively. The flow slides came to rest at post-failure slopes of 1° to 2°, after a runout of up to 100 m.

Related features were observed in coking coal stockpiles at northern Australian coal export terminals. The stockpiles were about 15 m high and failed within 10 to 20 seconds, reaching runouts of about 60 m. A laboratory testing program to investigate these events was undertaken by Eckersley (1990) who conducted a series of model tests to induce instrumented flow slides in a test tank. The stockpiles were modeled by placement of the moist coal in a tank 1 m high, sloping at 36° and a various average initial void ratios. The flow slides were then induced by slowly raising the water table in the model from behind the slope. The failure of the loose coal berms occurred in a period from 2 to 10 seconds at movement velocities about 1 m/s and attained post failure slopes as flat as 5°.The instrumentation revealed that the excess pore pressures were generated after the first movements and were a consequence of the collapse. As previously anticipated by Casagrande, loose saturated cohesionless materials could be brought to a state of undrained collapse by a drained trigger mechanism.

The experience of Eckersley is germane to the explanation of the failure of numerous coal mine waste dumps in British Columbia. Figure 3 summarizes data assembled by Dawson, Morgenstern and Gu (1992) that illustrate the relation between dump height and runout distance observed from failed coal mine waste dumps. Few of these failures are related to foundation weakness. Clearly, many failures have been experienced with substantial mobility. Fatalities have occurred and there have been significant environmental consequences.

The common belief prevails that there is insufficient saturation inside these waste dumps. They appear dry and natural particle segregation during dumping results in a free draining toe. However coal measure rocks are weak and readily break down during dump development. In addition somewhat finer material may be placed on occasion from the dump crest to form a blanket which will then be covered by subsequent dumping. Any finer layer inside the dump can be very loose, can retain water and develop pore pressures. Moreover its permeability can be sufficiently low to sustain the pore pressures necessary for the development of liquefaction flow slides.

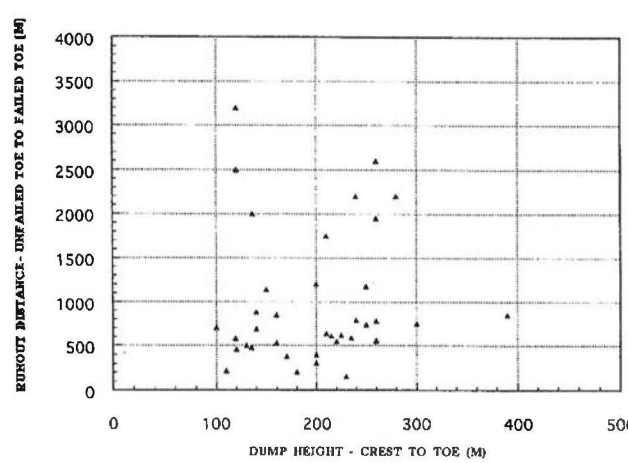

Fig. 3: Coal mine waste dump flow slide data (Dawson, Morgenstern and Gu,1992).

In summary:

1) Very loose cohesion less soils that are not entirely free-draining are disposed to liquefaction flow sliding.
2) Liquefaction flow slides are encountered in both natural and man-made deposits.
3) They are extremely destructive.
4) They can be initiated by both dynamic and static triggers.
5) Even fully drained trigger mechanisms can initiate undrained flow slides.

4. Undrained strength of loose cohesionless soils

The conceptual framework for understanding the factors influencing the undrained strength of loose saturated cohesion less soils was established by Casagrande in 1936. Bishop and Eldin (1950) demonstrated that under undrained conditions, contractant cohesionless soils behaved in a manner little different than contractant clays, and that they could be characterized in terms of undrained cohesive strengths.

Bjerrum et al. (1961) provided valuable data on the variation of undrained strength with consolidation pressure at a given void ratio and their experiments revealed that loose sands display an extremely brittle post-peak behaviour. Additional studies such as those of Kramer and Seed (1988) investigated other factors affecting the undrained strength of liquefiable cohesion less soils. The influence of initial anisotropic consolidation on the undrained strength is particularly noteworthy. If a very loose sand is consolidated under K_0 conditions, it may only be able to sustain a small additional increment of undrained shear before reaching its maximum resistance. The undrained shear strength is influenced by stress path, stress history, anisotropy and other factors. These details are beyond the scope of this presentation.

An important clarification of undrained strength behaviour was made by Castro (1969) and by Castro et al. (1982) through their introduction of the concept of steady state or ultimate resistance for the undrained strength of cohesion less soils. There are significant problems involved in determining the steady state strength in practice. Nevertheless, the concept that there is an ultimate resistance, independent of whether the soil was brought to failure by static or dynamic means, remains a basic building block in the mechanics of collapsible granular materials. Ishihara (1993) provides a valuable discussion on factors influencing the determination of steady state strength.

Conventional undrained strength tests provide a reasonable understanding of the undrained resistance of loose granular materials subjected to undrained monotonic loading. The steady state concept provides a basis for understanding the undrained resistance of liquefied sand in the flow state. One notes that it is finite, depending primarily upon the current void ratio, and that the resistance is independent of whether the material has been brought to the flow state by a static or dynamic trigger. A general flow liquefaction trigger criterion is needed in order to establish a comprehensive understanding.

5. The collapse surface

The concept of the collapse surface, introduced by Sladen et al. (1985b) to explain the Nerlerk subsea berm flow slides provides the missing link in practical terms. It is illustrated in Figure 4.

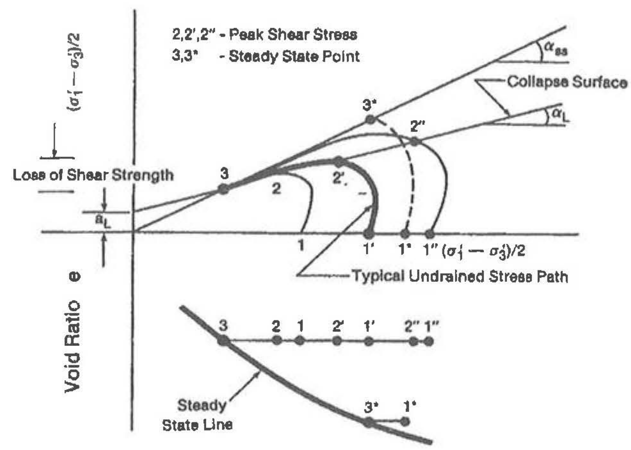

Fig. 4: The collapse surface (Sladen et. Al, 1986(b)).

The peaks of the undrained effective stress paths for sand samples at the same void ratio (e) but at various initial stress states, fall on a straight line in the p' - q (mean effective principal stress - deviatoric stress) plane and these stress paths all converge on the same ultimate or steady state. The line joining the peaks of the stress paths to the ultimate state is called the collapse line. The same material tested at different void ratios yields collapse lines of approximately the same slope but end at different ultimate state points. The integration of these lines in p'-q-e space creates a collapse surface that defines a state boundary surface at which the collapse of a very loose saturated material is initiated by undrained loading.

The collapse surface is nominally independent of stress path and constitutes a trigger criterion regardless of whether it is approached by monotonic or cyclic loading. This is a powerful concept unifying consideration of both static and earthquake induced liquefaction.

Experimental confirmation of the collapse surface concept and its generalization as a state boundary surface has been presented by Sasitharan et al., 1993 and 1994. The experiment illustrated in Figure 5 is particularly noteworthy.

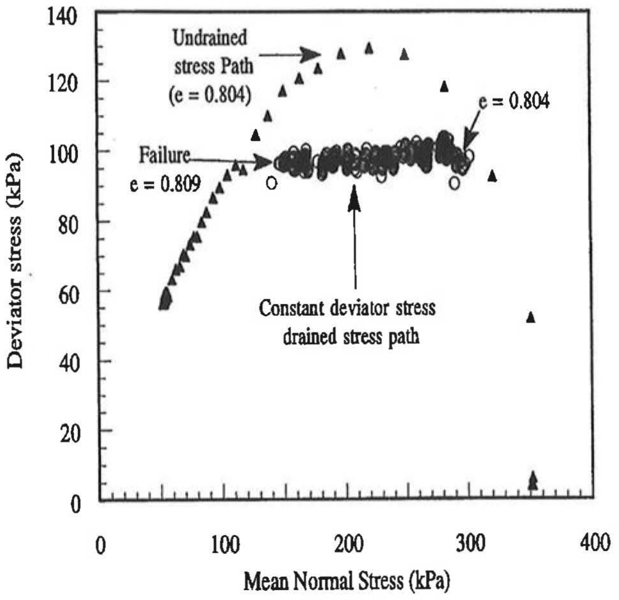

Fig. 5: Fully drained collapse (Sasitharan et al., 1993).

The undrained stress path of a loose saturated sand is shown in the p'-q plane. After peak, the stress path follows the collapse surface to the ultimate or steady state resistance. Another specimen at the same void ratio is consolidated to $p \approx 300$ kPa and $q \approx 100$ kPa. It is brought to failure under fully drained conditions by keeping the shear stress constant and by reducing p'. As soon as the stress point reaches the collapse surface, undrained failure occurs and the resistance falls along the collapse surface to the steady state. This test mimics closely the failure mechanisms that induced the liquefaction flow slides in the coal stockpiles in Australia and the coal waste dumps in Canada described previously. Recent tests indicate that collapse of coal wastes can occur even with void ratios as low as 0.4 -0.5. Typical inclinations of collapse surfaces are 20-25°. For pore pressures to be sustained during collapse of saturated soils, the permeability must be low enough to inhibit drainage. Field and laboratory data currently being accumulated suggest that in practice a permeability less than 1×10^{-2} cm/s will suffice, but further studies are needed to evaluate the simultaneous collapse and drainage of very loose deposits in order to refine this number.

6. Analysis of Collapse Mechanics

Neglect of the collapse surface as a trigger criterion can result in safety assessments that are very much on the unsafe side. Imagine the calculation of the Factor of Safety against deep seated instability of a waste dump that is apparently dry. In conventional terms this is a comparison of the available resistance with the resistances needed for equilibrium. As illustrated in Figure 6, this might imply a Factor of Safety of about 2. However the average mobilized stress may be close to the collapse surface. With a small rise in water pressure, possibly due to the development of a perched water table, the average effective normal stress will reduce, bringing the stress point to the collapse surface. Catastrophic liquefaction flow sliding could ensue.

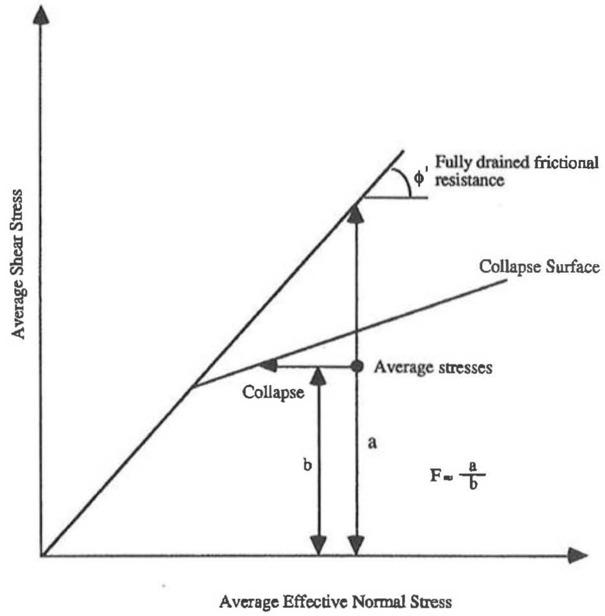

Fig. 6: Factor of Safety and collapsible materials.

Local collapse need not always result in flow sliding. If contained by non-collapsing material only limited deformations will develop. Finite element methods of analysis are needed to assess these conditions. Conventional stress analyses, static or dynamic, are applied to the soil mass until a stress point reaches the collapse surface. After this, post failure strain weakening along the collapse surface to steady strength is invoked and stresses are re-distributed in a progressive failure analysis. Details of these analyses have been published by Gu et al. (1993). The post-earthquake collapse of the Lower San Fernando Dam, Eckersley's model tests and other case histories have been successfully simulated in this way.

The collapse surface and steady-state concepts were used by Chillarige et al (1997) to explain the 1985 liquefaction flow slide that occurred in the Fraser River Delta, Canada. This was an example of what Terzaghi would have called "spontaneous liquefaction" and the analysis may be the first quantitative explanation of this process.

Bathymetric surveys revealed that the slide occurred at 10 m water depth between 27 June and 11 July 1985. The volume of the sediment involved in the flow slide was substantial (> 1 x 10^6 m^3). The bathymetric soundings provided contours both before and after the failure. Before the failure, the foreslope had a maximum angle of inclination of 23°. The post-event head scarp had a relatively steep slope with vertical relief of about 15 m and intersects basal planes that are subparallel to the original floor, dipping at angles of about 6°. An exaggerated section is shown in Figure 7.

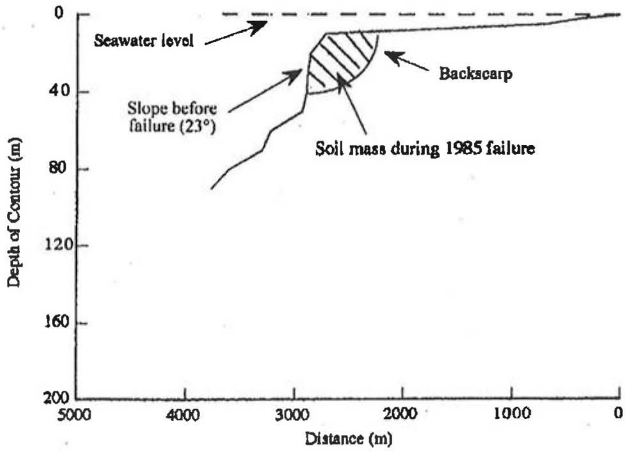

Fig. 7: Slope section (Chillarige et al.,1997)

The foreslope constitutes the extreme configuration of the slope before a liquefaction failure could occur. When flow liquefaction is initiated in the slope, the failure retrogresses towards its upper reaches, developing a flow slide. The retrogression continues until a stable or dense sand in the head scarp is reached.

An analysis of trigger effects excluded earthquakes, surface wave, rapid sedimentation or man-made effects as the proximate cause. Failure appeared to correspond with a period of low tides and it was shown that tidal drawdown in slightly gassy sediments creates stress paths that could bring elements in an unstable sand mass to the collapse surface. This trigger effect would generate retrogressive failure and hence "spontaneous liquefaction". The stress path calculated from the analysis is shown in Figure 8.

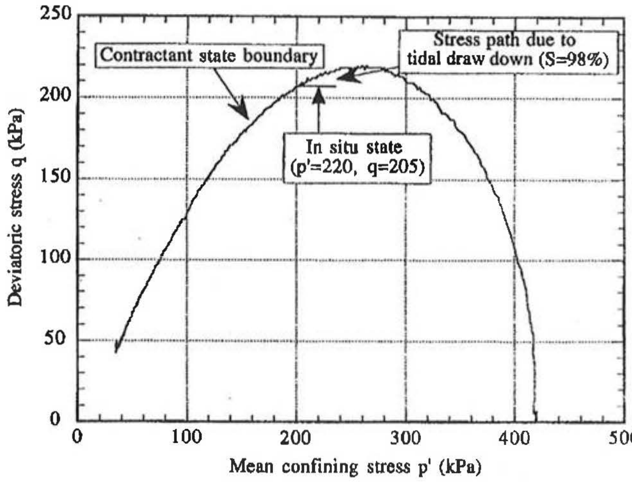

Fig. 8: Influence of tidal drawdown on triggering collapse (Chillarige et al., 1997)

7. Yielding of loose sands

Figure 9 shows the results of a constant shear stress compression test conducted with reducing normal stress on a dry loose sand. Collapse is identified at point B, mobilizing a resistance equivalent to the undrained collapse described in Figure 5. Both experiments are obviously evidence of the yielding of loose sand. While the collapse surface is useful for practical considerations, it is really a simplification of more general yielding of loose sands, cast within the framework of soil plasticity.

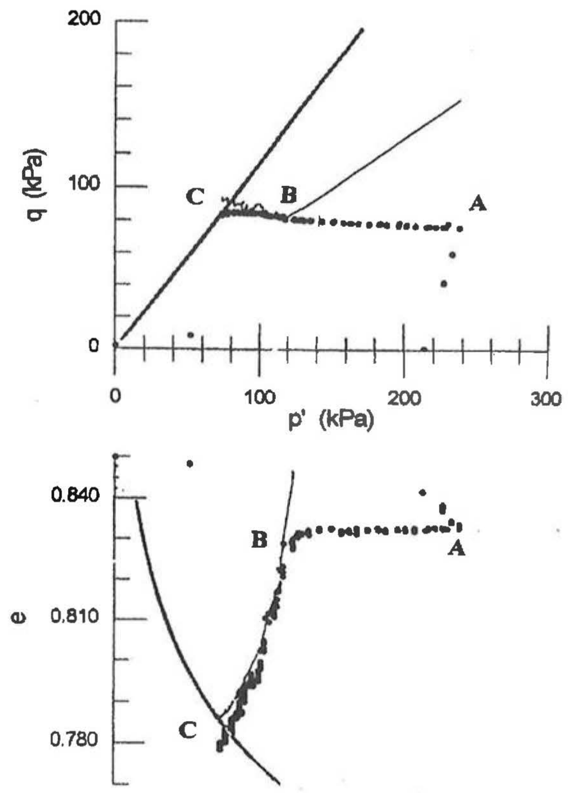

Fig. 9: Collapse of a dry sand (Skopek,1994).

A general theory of yielding of loose sands has been developed by Imam and is currently being prepared for publication (Imam et al., 2001). In this work it has been shown that the undrained effective stress path for loose sands can be used to determine the shape of the yield surface and to construct capped yield surfaces. The yield surface is highly anisotropic, hence accounting for the anisotropic response of loose sands to undrained failure. Loading itself need not be undrained. "Spontaneous liquefaction" occurs when the yield surface is reached and pore pressures cannot dissipate. Details are beyond the intent of this paper.

8. Conclusions

Liquefaction flow slides occur in both natural and man-made deposits. They are extremely hazardous and difficult to evaluate. Terzaghi had a clear understanding of the elements of the underlying mechanism in "Erdbaumechanik", and attributed their trigger to "spontaneous liquefaction".

This paper records progress in achieving a better understanding of "spontaneous liquefaction". The development of a number of concepts, and experimental investigations have been of value. They include:

- Undrained strength of loose sands
- Steady state behaviour and residual strength of sand
- The collapse surface
- Response of gassy sands to stress change
- Soil plasticity and the yield surface of loose sands.

These developments, together with advances in site characterization and applications to case histories, have contributed to an improved understanding of the initiation of liquefaction flow slides.

9. References

Berti, G., Villa, F., Dovera, D., Genevois, R., and Brauns, J., 1988. The disaster of Stava, northern Italy; in Hydraulic Fill Structures ed. By D.J. Van Zyl and S.G. Vick, ASCE, Special Geotechnical Publication No. 21, Fort Collins, p.492-510.

Bishop, A.W. and Eldin, G., 1950. Undrained triaxial tests on saturated sands and their significance in the general theory of shear strength. Geotechnique, Vol. 2, p. 13-32.

Bjerrum, L., Kringstad, S. and Kummeneje, O., 1961. The shear strength of fine sand. Proceedings 5th International Conference Soil Mechanics Foundation Engineering, Vol. 1, p. 29-37, Paris.

Casagrande, A., 1936. Characteristics of cohesion less soils affecting the stability of earth fills. Journal Boston Society of Civil Engineers, Vol. 23, p. 257-276.

Castro, G., 1969. Liquefaction of sands. Harvard Soil Mechanics Series No. 81, Harvard University, Cambridge.

Castro, G., Poulos, S.J., France, J.W., and Enos, J.L., 1982. Liquefaction induced by cyclic loading. Report by Geotechnical Engineers Inc. to National Science Foundation, Washington, D.C.

Chillariege, A.V., Morgenstern, N.R., Robertson, P.K. and Christian, H.A., 1997. Seabed instability due to flow liquefaction in the Fraser River Delta. Canadian Geotechnical Journal, Vol. 34, p. 520-533.

Dawson, R.F., Morgenstern, N.R., and Gu, W.H., 1992. Instability mechanisms initiating flow failure in mountainous mine waste dumps. Contract Report, Phase 1, to Energy, Mines and Resources Canada.

Eckersley, J.D., 1990. Instrumented laboratory flow slides. Geotechnique, Vol. 40, p. 489-502.

Gu, W.H., Morgenstern, N.R., and Robertson, P.K., 1993. Progressive failure of the Lower San Fernando Dam. Journal of the Geotechnical Engineering Division, ASCE, Vol. 119, p. 333-348.

Hutchinson, J.N., 1988. Morphological and geotechnical parameters of landslides in relation to geology and hydrogeology. Proceedings 5th International Symposium on Landslides, Vol. 1, p. 3-35, Lausanne.

Imam, S.M.R., Morgenstern, N.R., Robertson, P .K. and Chan, D.H., 2000. Yielding and flow liquefaction of loose sand. Soils and Foundations, submitted for publication.

Ishihara, K., 1993. Liquefaction and flow failure during earthquakes. Geotechnique, Vol. 43, p. 349-416.

Kramer, S.L., and Seed, H.B., 1988. Initiation of soil liquefaction under static loading conditions. Journal of the Geotechnical Engineering Division, ASCE, Vol. 114, p. 412-430.

McKenna, G.T., Luternauer, J.L. and Kostaschuk, R.A., 1992. Large-scale mass-wasting events on the Fraser River delta front near Sand Heads, British Columbia. Canadian Geotechnical Journal, Vol. 29, p.141-156.

Middlebrooks, T .A., 1942. Fort Peck slide. Trans. ASCE, Vol. 68, p. 723-742.

Morgenstern, N.R., 1967. Submarine slumping and the initiation of turbidity currents. Marine Geotechnique ed. By A.F. Richards, University of Illinois Press, p. 189-200.

Morgenstern, N.R., and Kupper, A.A.G., 1988. Hydraulic fill structures -a perspective. Hydraulic Fill Structures ed. by D.J. Van Zyl and S.G. Vick, ASCE, Special Geotechnical Publication No. 21, Fort Collins, p. 1-31.

Morgenstern, N.R., 1996. Geotechnics and mine waste management. Proceedings International Symposium on Seismic and Environmental Aspects of Dam Design, Santiago, Vol. 2, p. 5-26.

Morgenstern, N.R., 2000. Common Ground. Proceedings GEO-ENG 2000, Melbourne, Vol. 1, p. 1-20.

Müller, F., 1898. Das Wasserwesen der Provinz Zeeland, W. Ernst, Berlin.

Sasitharan, S., Robertson, P .K., Sego, D.C. and Morgenstern, N.R., 1993. Collapse behaviour of sand. Canadian Geotechnical Journal, Vol. 30, p. 569-577.

Sasitharan, S., Robertson, P .K., Sego, D.C. and Morgenstern, N.R., 1994. A state boundary surface for very loose sand and its practical implications. Canadian Geotechnical Journal, Vol. 31, p.321-334.

Seed, H.B., Seed, R.B., Harder, L.F. and Jong, H.L., 1988. Re-evaluation of the slide in the Lower San Fernando Dam in the earthquake of February 9, 1971. Report No. UCB/EERC-88/04, University of California, Berkeley.

Skopek, P., 1994. Collapse behavior of very loose sand. Ph.D. thesis, University of Alberta.

Sladen, J.A., D'Hollander, R.D. and Krahn, J., 1985 (a). Back analysis of the Nerlerk berm liquefaction slides. Canadian Geotechnical Journal, Vol. 22, p. 579-588.

Sladen, J.A., D'Hollander, R.D. and Krahn, J., 1985 (b). The liquefaction of sands; a collapse surface approach. Canadian Geotechnical Journal, Vol. 22, p. 564-578.

Terzaghi, K., 1925. Erdaumechanik. Franz Deuticke, Vienna.

Terzaghi, K., 1956. Varieties of submarine slope failures. Proceedings 8th Texas Conference on Soil Mechanics Foundation Engineering, University of Texas, Austin, p. 1-41.