THE AGE OF THE EARTH.

By Clarence King.

WITH PLATES I, II.

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ART. I.—*The Age of the Earth;* by CLARENCE KING. With Plates I and II.

AMONG the various attempts to estimate geological time none has offered a more attractive field for further development than Lord Kelvin's mode of limiting the earth's age from considerations of its probable rate of refrigeration, published in 1862.* At that time the consequences of his physical reasoning could not be fully applied to the conditions within the earth, so as to test the probability of his hypothetical case, for want of positive knowledge of certain properties of rocks, particularly the volume changes of melted rock in approaching and experiencing congelation, and the qualitative and quantitative effects of pressure upon the fusion and freezing points. Data then lacking are for the first time available, and with them it is proposed to apply a new criterion to the gradient of Lord Kelvin and to compare with it other cases of more probable earth-temperature distribution, which should have the effect of advancing his method of determining the earth's age to a further order of importance.

Accepting the hitherto unshaken results of Kelvin and G. H. Darwin, as to the tidal effective rigidity of the earth, and the further argument for rigidity advanced by Prof. S. Newcomb† from the data of the lately ascertained periodic variation of terrestrial latitude, as together warranting a firm belief in the rigid earth, it follows that solidity may be used as a criterion

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* Treatise on Natural Philosophy. Thomson and Tait, Part 2, Appendix D.

† Monthly Notices of the Royal Astronomical Society, vol. lii, No. 5, 1892.

to test the probable truth of many cases of earth-temperature distribution; at least so far as to justify the rejection of such as involve considerable liquidity of the upper couches. In an earth of which the superficial quarter of radius is composed of materials that contract from the fluid condition toward and in the act of congelation, any temperature gradient in which the downward heat augmentation exceeds the rate by which advancing pressure raises the fusion point, would obviously reach a fused couche, and all such distributions may be rejected as violating the requirements of rigidity.

A recent investigation of the rock diabase in its relations to heat and pressure offers the formerly lacking means of testing the admissibility of many cases of earth-temperature distribution from the point of view of solidity. Ten years ago in a laboratory established by me in connection with the United States Geological Survey, Dr. Carl Barus began a series of experimental researches tending toward the solution of some of the unknown but important points of geological physics. It has been my privilege to indicate the direction of much of the inquiry. The understanding between us maintained his entire independence in the mode and prosecution of the investigations and secured for him the fullest responsibility and credit for the purely physical results, many of which have at intervals appeared in this and other journals. For myself was reserved the privilege of making geological applications of the laboratory results. One of the most important of these is Dr. Barus's lately completed determination of the latent heat of fusion, specific heats melted and solid, and volume expansion between the solid and melted state, of the rock diabase.* To him I am also very generally indebted for aid in considering the present problem.

Diabase was chosen by me as fairly illustrative of the probable density and composition of the surface ·03 or ·04 of the earth's radius. For Laplace's law of distribution, density at the surface is taken at 2·75 and down one-tenth of radius at 3·88, yielding a mean density of the whole tenth of 3·33 and for the upper five-hundredths of about 3. For the whole tenth a rock like the extremely heavy basalt of Barenstein† (sp. gr. 3·35) would approach closely a fair mean expression of density. Typical hornblende-andesite comes closest to the average density at the surface, but diabase (sp. gr. 2·8 to 3·), nearly enough fills the conditions of the shell which this study seeks to investigate. The particular diabase under examination came from Jersey City, and was taken from the immediate vicinity of the Pennsylvania R. R cut.

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* This Journal, Dec., 1891, and Jan., 1892.

† J. Roth, Gesteins-Analysen, 1861, p. 46.

The following analysis is by G. W. Hawes:*

Silica	53·13
Alumina	13·74
Ferrous oxide	9·10
Ferric oxide	1·08
Manganous oxide	0·43
Lime	9·47
Magnesia	8·58
Soda	2·30
Potash	1·03
Ignition	0·90
	99·76

Astronomical and geodetic requirements make necessary that density should proceed downward in shells of successively greater value, but the surface density is 2·75 and the mean density of the whole earth is not twice that of diabase, whence it appears that no probable chemical distribution of material could result in a surface couche of ·05 of radius having a greater specific gravity than 3· to 3·3.

Waltershausen† in his interesting scheme of chemical distribution attempts to account for the augmentation of density chiefly by the increase of the heavy bases, but leaves the whole surface tenth of radius in silicates. Eruptions of alkaline earths or metals are unknown, in fact, with the exception of carbonates of superficial origin the whole visible body of the crust is of silicates, and the earliest rocks are seen to be made of the debris of still older ones. All that can be said is that there is absolutely no known reason why the surface tenth of radius may not be of silicates, nor why specific material of widely different thermal properties from diabase should be postulated.

The two principal conditions within the interior of the earth upon which physical state and all purely physical reactions of the specific materials depend, are the distributions from center to surface of pressure and heat. Secular or sudden variations of either or both have the power, if carried sufficiently far, to disturb chemical and physical equilibrium and produce changes of volume, rigidity, viscosity, and conductivity, as well as changes of state from liquidity to solidity, and the reverse. Before proceeding to consider in detail some of the results of heat and pressure as existing in the surface ·05 of radius, it is desirable to glance at the relations of these two great antagonistic

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* This Journal, III, vol. ix.

† "Rocks of Sicily and Iceland."

energies in the whole radius. Plate I gives earth-pressures from Laplace's law expressed in a gradient of which the ordinates are 100,000 atm. (larger divisions 1,000,000 atm.) and the abscissæ tenths of radius. Upon the same diagram are delineated two hypothetical cases of earth-temperature, the abscissæ remaining as for the pressure line, tenths of radius, and the ordinates corresponding in interval to the 100,000 atm. lines, are taken as each 1,000° C. The left vertical boundary of the plate represents the center of the earth and the right one the surface. The upper heat gradient corresponding to a temperature of 3,900° C. at the earth's center is the 100×10^6 curve of Kelvin. The lower is computed for a central temperature of 1,741° C., about the melting point of platinum, and a secular cooling in 20×10^6 years. Data for the construction of these gradients are given in the tables a few paragraphs later. The feature here called attention to is the exceedingly slight change of temperature from very near the surface downward to the center. In the Kelvin gradient even after the lapse of 100×10^6 years the original maximum temperature is reached within ·05 of radius and remains thence unchanged to the center. Pressure, on the other hand, augments with one downward sweep through the entire radius. On Plate I its line is seen cutting both temperature gradients near the surface, passing the 1,741° C. line at a pressure of 175,000 atm., and the Kelvin line at 390,000 atm.; thence steadily augmenting until at the center it reaches the impressive figure of 3,018,000 atmospheres.

Since we are to look to heat and pressure for the keys to the physical condition of the matter of the earth, it is important to realize from the relation of these gradients, first, that the great effect of heat in opposing and overcoming the results of pressure must be limited to superficial earth-depths not exceeding 200 miles for an earth of the Kelvin assumptions; secondly, that below this depth and onward to the center there is a complete reversal of relations and a great and continual increase of pressure available to oppose and destroy the volumetric and other molecular effects of a temperature which has ceased to increase. The empire of heat over pressure is thus seen to be purely superficial, while that of pressure over heat begins not far below the surface and extends more and more powerfully to the center. This is obviously true only for such moderate assumptions of heat and time as are given in the gradients on Plate I, but it will be shown later that these figures are, upon the criterion of solidity, far more probable than very hot or very old earths.

Out of the infinite number of possible earth-temperature gradients, to discriminate the probably true case, is of critical importance in any attempt to determine the earth's therma age or to delimit the period of active geological dynamics.

Pressure and Temperature Tables.

The following tables offer figures for the construction of th pressure and some of the temperature gradients on both Plate I and II. Data for the distribution of earth-pressure may b obtained either from the formula of Laplace or that of G. H Darwin for radial earth density, combined with the know decrease of terrestrial gravitation from center to surface.

In table 1, Laplace's law is used as giving the most conserva tive values of density at great depths. For the superficial ·2 o radius, however, the two density laws are near together, an as the thermal phenomena which determine the earth's ag are probably wholly in the surface tenth, either law migh be applicable to the present purpose. As, however, Darwin' law requires a surface density of 3·7, while Laplace only 2·75 the latter accords better with the average specific gravity o superficial rocks and is, therefore, here preferred.

Tables 2, 3 and 4 give data for three temperature gradient derived by mechanical quadrature from the well known Fourie equation in the manner given by Lord Kelvin, and are con sidered as sufficient in number and variety to indicate th character of the data; figures for the other gradients shown o Plate II are therefore omitted.

Table 2 presents data for the Kelvin gradient, 3,900° C initial excess, surface rate ·03600 in degrees Centigrade pe meter of depth, and secular cooling 100x10^6 years. Eart temperatures in ° C. are given for depths that are expresse both in miles and fractions of radius and extend to 250 mile or about ·06 of radius. Surface rate appears both in ° Fah and feet, and ° C. and meters. Tables 3 and 4 exhibit simila data for earths of lower initial excess and shorter periods o secular cooling. Table 3 is computed for an earth of 1,740° C 20x10^6 secular cooling, and table 4 for 1,230° C., 10x10^6 coo ing.

TABLE NO. 1.

Estimated Earth Pressures (Laplace's densities) *n* being radial distances from t center of the earth and *p* being the pressure corresponding to *n* expressed in atmospheres.

n. Earth Rad.	*p.* Atm.	*n.* Earth Rad.	*p.* Atm.	*n.* Earth Rad.	*p.* Atm.
1·000	0	·94	116000	·5	1680000
·995	8600	·92	162000	·4	2100000
·990	17400	·90	199000	·3	2470000
·985	26400	·80	497000	·2	2770000
·980	35600	·70	852000	·1	2950000
·960	74500	·60	1260000	0	3020000

Table No. 2.

Estimated Earth Temperatures. Initial excess of 3900° C. 100 millions of years secular cooling with surface rate of 1° F. for 50·6 feet of depth. Thermal conduction 400 ft^2/year (Lord Kelvin's case).

Miles deep.	Rate in ° F and ft.	Rate in ° C and meters.	Temperature in ° C.	Depth in earth radius.
0	·01977	·03600	0	·00000
12	·01924	·03510	726	·00312
25	·01773	·03230	1412	·00625
50	·01279	·02330	2543	·01250
75	·00742	·01350	3275	·01875
100	·00346	·00630	3658	·02500
125	·00129	·00236	3825	·03125
150	·00039	·00071	3881	·03750
175	·00009	·00017	3897	·04375
200	·00002	·00003	3901	·05000
225	·00000	·00001	3902	·05625
250	·00000	·00000	3902	·06250

Table No. 3.

Estimated Earth Temperatures. Initial excess 1741° C. (about melting point of platinum) 20 millions of years secular cooling with surface rate of 1° F. to 50·6 feet of depth. Thermal conduction 400 ft^2/year.

Miles deep.	Rate ° F and feet.	Temperature ° C.	Depth in earth radius.
0	·01977	0	·00000
6	-----	359	·00156
12	·01726	693	·00312
25	·01147	1218	·00625
37	·00581	1534	·00937
50	·00224	1675	·01250
62	·00066	1725	·01562
75	·00015	1738	·01875
87	·00002	1741	·02187
100	·00000	1741	·02500

Table No. 4.

Estimated Earth Temperatures. Initial excess 1230° C., 10 millions of years secular cooling with surface rate of 1° F. to 50·6 feet of depth. Thermal conduction 400 ft^2/year.

Miles deep.	Rate ° F. and feet.	Temperature ° C.	Depth in earth radius.
0	·01977	0	·00000
12	·01506	662	·00312
25	·00665	1063	·00625
37	·00171	1198	·00937
50	·00025	1227	·01250
62	·00002	1230	·01562
75	0	1231	·01875
87	0	1231	·02187
100	0	1231	·02500

TABLE NO. 5.

Estimated Melting Point and Depth for the rock Diabase expressed in radial earth distance, pressure and melting temperature.

n. Earth Rad.	p. Atm.	θ_m. ° C.	n. Earth Rad.	p. Atm.	θ_m. ° C.
1·000	0	1170	·920	162000	5210
·995	8600	1380	·900	199000	6100
·990	17400	1600	·8	497000	14000
·985	26400	1830	·6	1260000	33000
·980	35600	2060	·4	2100000	54000
·960	74500	3030	·2	2770000	71000
·940	116400	4080	·0	3020000	76000

Table 5 contains a prolongation of Barus's line of melting point and depth for the rock diabase, expressed in radial earth distance n, pressure p (Laplace's densities), and melting temperatures, θ_m.

The Chart.

The chart constituting Plate II is constructed to present the passage of certain hypothetical temperature gradients through the uppermost ·08 of the earth's radius, and the position in the same field of Barus's line marking the melting point in depth of diabase, thus defining the relations of the various distributions of earth-temperature to liquidity. The value of the ordinates is each one thousand degrees Centigrade ; the abscissæ, which are platted as equal in length to the ordinates, represent hundredths of radius counting downwards from the surface which is indicated by the right vertical boundary of the chart.

Kelvin's application of the Fourier equation involves an assumed initial excess of temperature, an assigned value of rock conductivity, a given period of secular cooling and the surface rate of augmentation of earth-temperature. As thus applied to the case of the cooling earth, it is obvious that while the body was of uniform initial heat there would be no augmentation of temperature from the surface downward, or otherwise expressed, the surface rate would be ∞ ; but the moment refrigeration began a finite rate of downward increment would be established. Since the earth's surface is represented on the chart by the right vertical boundary, that line would be the thermal distribution for the rate ∞. A complete process of refrigeration would cause the rate to decline until the earth reaches the temperature of space and the line of initial tangency coincides with radius, and the rate 0. The angular relation of the initial tangent of the present as compared with that of the rate ∞ is determined from observed surface augmentation.

The value of the integral and the surface rate for any gradient does not change if conductivity and age vary reciprocally, and the surface rate does not change if the initial excess of temperature varies at the same rate as the square root of the product of conductivity and the time of secular cooling. If the square root of the product of conductivity and age be increased any number of times and the depth also be increased the same number of times, temperature remains unchanged if the initial excess is unchanged, but if the initial excess changes, temperature will change in the same ratio.

Upon the chart are delineated two families of temperature distributions. Those in continuous lines, lettered *a* to *f*, are calculated in accordance with the maximum surface rate of 50·6 ft. to 1° Fahr., being the generally accepted rate at the time Kelvin's curve was published. Those in dotted line and lettered *g* to *i*, are constructed for the rate of 75 ft. to 1° Fahr. the smallest of the observed inland rates. It is the value given by Hallock* for the recently completed boring near Wheeling, W. Va. The last published value as reduced from all available data by the B. A. committee is 64 feet to 1° Fahr. It is, therefore, extremely probable that unless some general but unrecognized cause, like a variation of temperature due to the chemical action of hot water and progressive downward either with heat or pressure, tends to raise or lower the mean rate, the true surface distribution falls between the values of 50·6 and 75 ft. per ° Fahr. upon which the two families of gradients are based.

The diabase line for melting temperature and depth D D is traced from its superficial fusion point, 1,170° C., downward according to the law established by Dr. Barus and expressed in table 5. This is the special point of interest in the chart and in the conclusions to which it gives rise. In passing from this surface value of 1,170° C. through ·1 of the radius, the fusion temperature is raised to 6,139° C.; continuing thence to the center of the earth it reaches the surprising value of 76,200° C. In consequence in an earth all of diabase any temperature gradient having an initial excess of less than the above central value must in reaching the surface either intersect the line D D twice or fall wholly beneath it. Since this line represents melting temperature, any point vertically above it in the chart is necessarily more highly heated than the melting temperature for the same depth, and hence in fusion. Conversely, any point below the diabase line being below the melting temperature for that pressure and depth, falls into solidity. Thus

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* This Journal. vol. xliii. p. 234, 1892.

the chart is divided into two areas by the line, that above it representing fluidity, and that below, solidity. For a diabase earth to have been wholly melted an initial excess of 76,200° C. would be required. Obviously any earth having an initial excess of less than the surface melting point 1,170° C. would have been always completely solid. Any initial excess above that figure and below 76,200° C. requires at the moment before refrigeration began, a solid nucleus and a fused zone above it extending to the surface, and the lower the initial excess the larger the solid nucleus of compression and shallower the initial couche of surface fusion. Knowing the law of the rise of the fusion-point it is a simple matter of computation to determine, for any assumed initial excess, exactly the radial value of the original solid nucleus and of the original supernatant fluid couche. For the region covered by the chart these values may be directly scaled off.

Fourier's equation enables us to go further and assuming that refrigeration is the result of conduction alone to determine the temperature distribution for any given period of refrigeration, and what is of particular geological interest the rate at which the fused couche is encroached upon by an overlying superficial crust of congelation, and the existence, depth, temperature and pressure differences of any residual fluid couche between the upper and lower solids. The relation, therefore, of any temperature gradient to the diabase line offers an immediate test of its admissibility as a probable case. Any temperature gradient that in passing across the area of the chart from below to the surface intersects the diabase line must in reaching the low temperatures of the surface intersect it again, and the zone included between the pair of points of intersection being above the line, and hence for that interval of radius above the fusion temperatures, must be a melted shell, and as on the criterion of solidity the existence of any considerable fusion is precluded, such a case of temperature distribution may be rejected.

I will now trace the several temperature distributions upon the chart, and note their relations to time and solidity, beginning with the family delineated in continuous lines (surface rate 50·6 ft. to 1° Fahr.). Line *b*, the gradient of Kelvin, 3,900° C. initial excess, 100×10^6 years secular cooling, is seen to enter the chart from the lower regions, maintaining even to the shallow depth of 226 miles from the surface, practically, its original maximum temperature. From the center of the earth up to this point it has remained in the initial solid of compression. At the depth noted it intersects the diabase line and passes into fusion. Since almost the full initial temperature is maintained up to its intersection, it follows that that depth nearly marks the original surface of the solid nucleus and that the distance of 226 miles thence to the surface measures the depth of the original couche of fusion. Following the gradient toward the surface it is seen after describing its great convexity in the fluid region to intersect the diabase line a second time and enter a congealed shell or crust formed by cooling a surface portion of the initial fused couche, and leaving between the nucleus and crust, a residual present shell of fusion of 200 miles from top to bottom. The obvious tidal instability of a 26-mile crust resting upon 200 miles of truly fluid magma is sufficient basis for the rejection of this particular case of temperature distribution. To fulfill the requirements of rigidity either the time of cooling must be vastly greater to admit of entire congelation, or the initial excess materially less.

As an illustration of the first of these alternatives, gradient *c* with the same initial excess as *b* (3900° C.) has been developed to complete solidity which on computation proves to have required about 600×10^6 years, at which time it has but just reached tangency with the diabase line. Yet we are absolutely precluded from accepting it as a probable case and assigning 600×10^6 years as the age of the earth, because the temperature values of its emergence at the surface fall below even the 75 feet to 1° Fahr. surface rate. Its emergence is at a rate of ·0081° Fahr. per ft. (124 ft. per ° Fahr.) which is far less than the (Hallock) rate used in the dotted gradients, itself much less than the accepted mean rate of the British Association Committee.

Gradient *d*, 1,950° C. initial excess, and 15×10^6 years secular cooling, falls still some millions of years short of solidity. The initially fused surface couche was about 66 miles in depth, the present crust 33 miles thick and the present residual fluidity of 33 miles depth from top to bottom. Here again the liquid zone involves tidal instability and requires the rejection of the line.

Gradient *e* offers more satisfactory conditions: with an initial excess of 1,750° C., about the normal melting point of platinum, and an age of 20×10^6 years, a condition is reached which throws the convexity of the gradient below the diabase line in complete solidity and fulfills all the conditions. Here then is a possible age for an earth of diabase. Its initial surface couche of fusion would have been about 53 miles and is now wholly cooled into solid crust and united with the original solid nucleus of compression.

Gradient *f*, initial excess 1,230° C. and 10×10^6 years secular cooling, would in its first stage have shown only about five or six miles of surface fusion which would very shortly have cooled into solidity.

For those whose interest centers in earths of great age and high temperature, gradient a is given, initial excess 7,800° C. and 400x10^6 years secular cooling. This has not been projected to the deep, but would not reach solidity until over 1,500x10^6 years, a truly uniformitarian specimen.

Turning now to the family of three gradients in dotted line, computed to conform to the surface rate of 75 feet to 1° Fahr., the first, g is seen to be of the same initial excess as the Kelvin 3,900° C. line. In spite of its long cooling even after 237x10^6 years it is still very far from solidity. Of the original fluid couche of 226 miles, only about 60 miles has been congealed into crust, 166 miles remaining fused.

Gradient h, initial excess 2,560° C., and a 100x10^6 years refrigeration has an original fluid couche of 120 miles with a present crust of 56 miles and an existing residual couche of fusion of 64 miles, a case also inadmissible from the point of view of instability.

Gradient i, initial excess 1,760° C. (platinum melting point), and 46x10^6 years of cooling, had originally a 53-mile surface couche of fusion which has long since passed into solidity. The following table sums up the condition of all the gradients as to initial excess, initial depth of surface fusion, time of cooling, thickness of crust congealed and present residual couche of fusion.

TABLE 6.—LIQUID SOLID CONDITIONS FOR DIABASE EARTH.

A.—Gradients having the surface rate of 50·6 to 1° Fahr.

Initial excess. ° C.	Initial depth of surface fusion. Miles.	Time of cooling. Years.	Thickness of crust congealed. Miles.	Residual couche of fusion. Miles.
3900	226	100x10^6	26	200
1950	66	15x10^6	32	33
1741	53	20x10^6	crust and nucleus united	0
1230	6	10x10^6	“ “	0

B.—Gradients having the surface rate of 75 feet to 1° Fahr.

3900	226	237x10^6	50	166
2560	120	100x10^6	56	64
1741	53	46x10^6	crust and nucleus united	0

Comparison of gradients of equal initial excess and successively longer periods of secular cooling shows the ratio of their retreat from right to left across the chart or from lower to higher values of depth and time.

With each augmentation of age the initial tangent defining the surface rate is seen to have declined further and further from the original rate α, coinciding with and passing first the maximum, then the minimum rate thence declining into the region of inadmissible rates.

The probable conditions of the true gradient are as to initial excess and age such as fall below the diabase line into solidity and emerge at the surface with a rate which has not declined below the mean (B A) rate of 64 ft. to 1° F. From the point of view of solidity no gradient of initial excess above 2,000° C. is admissible: that of 2,560° C., even after 100×10^6 years cooling still shows a deep shell of fusion (sixty-four miles from top to bottom), and since it emerges on the minimum rate it has already fallen below the admissible tangent.

Gradient *d*, 1,950° C. and 15×10^6 years, just cooled to the maximum surface rate has still an inadmissible fluid shell, but if refrigeration had been continued for 7×10^6 to 9×10^6 years more the line would have fallen below the solidity line and its surface rate would not have passed the mean value. Hence a 1,950° 24×10^6 year earth is possible and marks about the superior limit admissible for initial excess.

From the point of view of age no greater time of cooling is allowable than enough to bring the gradient for any initial excess to the mean surface rate. Thus the condition for excess and age exclude a line of over 2,000° C. and 24×10^6 years. Conductivity remaining of the value used, any higher excess involves fluidity, and any greater age an inadmissible surface rate.

To the extent, therefore, that solidity is a valid criterion and so far as the melting temperature of diabase may be supposed to apply to the depth examined, there is no escape from an earth of the low age and temperature given except by impugning the rate of surface augmentation and the value of rock conductivity here employed.

Whoever has examined the B. A. committee's reports and summaries on underground temperatures must have realized the obstacles to the evaluation of a true mean rate. The range of observations is wide, from high rates due to residual vulcanism to low ones produced by neighboring bodies of cold water, such as are described by Wheeler from mines near Lake Superior.* It is not, however, likely that by rejecting anomalies and assigning probable weight to further observations the present value will be moved to an important extent.

We have seen that all probable distributions of earth-temperature involve in the initial stage a great solid nucleus, practically the whole body of the earth, with a shallow surface shell of fusion. In the case of the 1,741° C., 20×10^6 year earth there was an initial melted shell of 53 miles. Obvi-

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* This Journal, vol. xxxii, 1886.

ously it cannot be correct to apply the rock-conductivity value obtained at air temperatures and normal pressure to even so young and cool an earth with its couche of an initial temperature of 1,741° C. and a pressure difference between the top and bottom of 22,000 atm. The probable method of cooling the couche into solidity, involves three corrections of the accepted rate of refrigeration: *a*, acceleration of the process by possible convection; *b*, the direct effect of heat and pressure upon conductivity, and *c*, the relative conductivity of matter at the same temperature, liquid and solid.

a. Convection. Leaving out of present consideration possible polymerization of the magma, or the descent of solid bodies of crust, vertical transfers of the liquid matter in the fused couche depend upon differences of density and this upon the ratio of the rates at which density is raised by pressure and lowered by heat. Isometrics of melted rock under high pressure are of course beyond the reach of experimentation, hence we are forced to look to those of the available materials. Isometrics from high pressure observations have been found to slope as follows:

Ether	8·7	atm.	per ° C.
Alcohol	10·5	"	"
Thymol	13·9	"	"
Dyphenylamine	15·4	"	"
Paratoluidine	13·9	"	"
Glass, computed	10·0	"	"

Since ether boils at 34° C. and dyphenylamine at 310° C., the range here given is wide. It is reasonable, therefore, to take the mean value, 12·5 atm. per ° C., as an index of the slope sought for. In the Kelvin earth this rate occurs between ·010 and ·015 of radius, the crust being ·0065 of radius thick. In so far, therefore, as the isometrics can be regarded as parallel straight lines with a slope of the order of the value given above, convection can only have taken place in the first 52 miles of the initial couche of fusion and in the present residual couche of 200 miles only the upper 26 miles would be subject to convection. In younger earths the above value per ° C. will be found much nearer the surface so that in them convection would be confined to a shell which is shallow in proportion as the earth is young. Initially when the whole earth was at one temperature there could have been no convection, since the change of temperature in depth was *nil*, but the change of density due to pressure was always pronounced. In the case of the 1,741° C. earth the zone of convection would have early been covered and extinguished by the thickening crust and therefore would have played no very important part in accelerating the loss of heat, and thus for this particular initial excess is of small effect in shortening the estimate of earth's age.

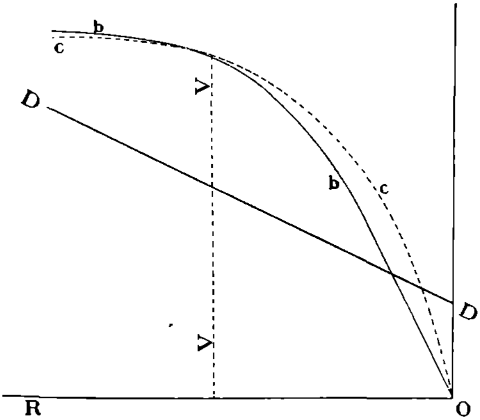
b. The direct effects of heat and pressure upon the conductivity of matter under such high temperatures and pressures are also beyond laboratory investigation, and again we are driven to use the determined conductivity value unmodified, or seek for some other property which may be considered as its approximate measure. Such an index is found in viscosity which if not of high quantitative significance in defining the changing values of terrestrial conductivity in depth, nevertheless affords data applicable at least for determining the sign of an important correction.

Dr. Barus has lately determined that at least 200 atmospheres of pressure are required per one degree Centigrade in order that viscosity may remain constant. Examining several temperature distributions of the chart and applying the computed augmentation of earth-pressure, it appears that the required relation (200 atm. to 1° C.) is found at successively lower depths for successively higher values of initial excess and age. In the 1741° C. case the relation after 20x10^6 years' cooling is found at about ·016 of radius counting from the surface, where the vertical broken line *v. v.* of the chart intersects the gradient and marks the locus of stationary viscosity. As above this point temperature relatively to pressure has augmented more rapidly than the ratio required for constant viscosity it follows that viscosity has been diminished by temperature more than it has been raised by pressure. Below the stationary point, on the other hand, an excess of pressure above the required ratio is available for increase of viscosity.

For the gradient of 3900° C. excess the transitional depth is indicated by the intersection of the broken line V V. In both cases the transitional points occupy positions in their respective gradients not far below their full initial temperatures, and pressure having been most stationary the transitional points have moved but little during the whole period of secular cooling, and the earth shells passing through them have divided radius into a lower solid of higher viscosity and a surface couche, partly liquid partly solid of lower viscosity. So far therefore as viscosity indicates the behavior of conductivity, that also should have been systematically diminished (relatively to the surface value obtained at normal pressure and temperatures and used in the construction of the gradients) from the surface downward for a small fraction of radius, till at the appropriate depth for each excess and age of cooling, it reaches a transitional value and thence increases.

How this correction, of at present unknown value, affects the coördinates of a given gradient qualitatively, is shown by the following figure in which are given the diabase melting point and pressure line, D D, gradient *b* of 3900° C. excess and 100x10^6 year's cooling, with the viscosity transitional line V V intersecting it, also a dotted line *c*, indicating the position of the *b* gradient corrected for diminished conductivity (viscosity).

Lagging to the right of the uncorrected gradient obviously the dotted line would require longer refrigeration to reach the state of solidity, and it is equally important to note that its position requires its emergence at the surface with a higher rate than the uncorrected line and thus extends the time of cooling down to the mean rate which marks for all gradients the present limit of the process.



c. Liquid-solid conductivity. Closely involved in the above heat-pressure-viscosity correction is the change of conductivity on passing isothermally from solid to liquid. Here again the results of Dr. Barus* throw important light.

The relatively higher thermometric conductivity of the solid over the liquid of equal temperature indicates an additional plus correction for time values.

Both the minus correction due to convection, and the plus corrections based upon conductivity diminished below the Everett figures, sink in importance as we pass from earths of higher to those of lower initial excess, so that until some approximate quantitative values can be given them we have no warrant for extending the earth's age beyond 24 millions of years.

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* The change of heat conductivity on passing isothermally from solid to liquid. C. Barus, this Journal, July, 1892.

That the application of the criterion of solidity here made to Kelvin's method is open to the objection of being based on the physical relations of an extremely superficial fraction of radius is obviously true. Ignorance of the deeper, interior distribution of specific materials and of their relations to the degree of heat and the range of pressure to which they are subjected, forbids the construction of a generalized line of melting temperatures for the whole of radius.

It might, therefore, be contended that a reversal of the diabase conditions is possible, and the deeper materials may possess the property of ice-fusion; their melting temperatures suffering depression instead of elevation. The high densities required in lower earth-depths have constantly suggested the concentration there of heavy metals and the examples of meteorites has further influenced the idea of a metallic nucleus chiefly of iron. And as iron at normal pressure unmistakably exhibits ice-fusion, any great iron mass at the center might be supposed to exist as a liquid in spite of the enormous pressure there exerted.

The distribution of materials and of "state" under this assumption involves a metallic (iron) nucleus, liquid from ice-fusion, overlaid by less dense couches which at some unassignable depth pass into silicates of the diabase type, solid from compression under the law shown for diabase, and solid to the surface as required by tidal effective rigidity.

Ice-fusion, however, is an exceptional phenomenon, nor have we any but the most limited data as to its range as regards temperature and pressure. Iron is conceded to contract in the act of fusion, but cold iron is more dense than the substance either just above or just below the fusion point. It is not beyond the range of probability that excessive pressures might bring about the same density in iron that cooling does, and thus isothermally convert ice-fusion into the normal type and produce a solid nucleus. However that may be, tidal effective rigidity excludes fusion of either type for at least ·2 of radius.

Other methods have been used for obtaining a measure of the earth's age or for some definite portion of geological time.

Earth-Age from Tidal Retardation.

Kelvin's comparison of the earth's present figure with that of a thousand millions of years ago when the terrestrial day would have been only half its present length is one of the most interesting. The earth, if then plastic, would have yielded to four times the present centrifugal force at the equator and shown a correspondingly greater flattening at the poles and bulging at the equator and "therefore" (as Tait expresses it) "as its rate of rotation is undoubtedly becoming slower and slower it cannot have been many millions of years back when it became solid, else it would have solidified very much flatter than we find it." This implies that because a computed earlier and greater value of ellipticity does not now exist it could never have existed, in other words, that terrestrial rigidity has been and is of such value that a form taken in the remote past by the solid earth would not be modified by the tidal retardation of rotation and its attendant change of centrifugal force.

There is in modern geology a growing body of evidence which is believed to prove the very general plasticity of the lithosphere, by which it may experience important deformations from very *slowly* applied stresses. So strongly has this belief taken root that many American geologists accept "isostasy" and consider it to be an expression of a fluid equilibrium for the earth.

From abundant geological observation plasticity must be admitted for slow deformations enormously in excess of the small change of figure which the stress of tidal attraction would produce but for elastic resistance.

Although rigidity prevents a sudden tidal deformation of five feet it does not prevent a slow radial deformation of five miles of the surface matter. How then can it be supposed to resist the slow change of stress due to tidal retardation of rotation? The excess of the equatorial over the polar axis is now roughly 25 miles while the radial range of surface inequalities of the lithosphere is about 12 miles, of which a large part dates from this side of the beginning of Tertiary time. If past plasticity equals present values, the earth's figure could never have been a survival from some assumed earlier epoch when centrifugal force was greater, but must always have been a function of the slowly diminishing rate of rotation.

If the conclusions of the earlier portion of this paper are true they go further and exclude the idea of a formerly fluid earth and *any* epoch of solidification. With any admissible assumption of initial excess nearly the whole earth must have been solid from the date of the first collocation of its matter.

To whatever radial depth plasticity may descend, what is enough for geologically recent superficial inequalities is sufficient for adjusting the figure of the earth to existing forces of rotation.

The same coast lines which remain stationary under tidal stress are slowly rising and falling in a hundred places under the slow application of subterranean energy.

It therefore appears that no time measure can be deduced from the supposed fixing of the present ellipticity at some past date.

Astronomical Measure of Earth-Time.

Croll's hypothesis from which it was proposed to fix dates by secular variations of eccentricity and to correlate the *climatic* effects of those variations with geological operations and thus measure certain intervals of geological time, required so much questionable physical geography and left so many physical doubts that few have been found to accept the excessively complex chain of effects lying between eccentricity data and geological facts. The objections of Professor Newcomb, noticed rather than answered, left Croll's doctrine where it was permissible to believe that there was *something* in it, but not necessarily that definite sequence of climates which is the core of the idea.

The gap in Croll's scheme seems to have been successfully stopped by Sir Robert Ball whose interesting proof of the seasonal inequality of the thermal element in climate due to position of the equinoxes, and its intensification in periods of high eccentricity offers a new hope for the accurate dating of at least very modern geological climates. From this point of view late geological history requires reëxamination, and if it should appear that a sequence of climates has existed closely paralleling the thermal variations which the astronomical values seem to afford, an extremely probable case will have been made out. And this case would be practically substantiated if the hypothesis of H. Blytt should yield the confirmation for which he hopes. Blytt* proposes and has already attempted to correlate the secular *attractional* changes due to varying eccentricity and precession with the observed successive shifting of beach lines.

So far as he has proceeded it is of interest to note that his time estimates are more in harmony with the physical than the stratigraphical figures.

Periodic changes in the figure of the hydrosphere relatively to the solid earth, due to alterations of attraction, might be predicted with some confidence if it were clear that the lithosphere would under the slow stresses involved continue to exercise a degree of rigid resistance comparable with that it opposes to the tidal stress, but there is no proof that it would.

Since we find the solid earth undergoing slow deformations to-day which are relatively permanent, while its effective

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* The probable cause of the displacement of beach lines. H. Blytt—1889, Christiania Videnskabs Forhandlinger No. 1—Additional note 1889—second additional note 1889.

elastic resistance to tidal stress is sufficient to permit a water tide, it appears that either the purely telluric stresses are greater than the moon's attraction, or that there is for the time rate of application of equal stress, a transitional value above which the elastic resistance of the earth-solid is enough to conserve figure, and below which plastic deformation is easy; a relation of properties such as Kelvin suggests for ether. Under the former alternative, deformations due to purely telluric forces might by upheaval or subsidence at any time mask or counteract astronomical beach shifting. In the latter case to make use of the astronomical data for displacement of beaches, it is required to ascertain the time rate of terrestrial plasticity accurately enough to know that relatively to the duration of eccentricity and precession cycles and their correlative attractional variations, the reaction of the lithosphere would differ enough from that of the hydrosphere to allow of the beach shifting sought.

Beyond the most modern geological dates the grander earth deformations have carried ancient beach lines out of all recognizable radial relations with each other and the several oceans of which they mark the shores, or else as is frequently the case with rising continents they have been wholly effaced by erosion. Evidently the Croll-Blytt time measure, interesting as it may prove to be for recent dates, is at present inapplicable to any general determination of the earth's age.

Earth-age measured by Sun-age.

Since the incrustment of the earth would be almost immediately followed by a climate controlled wholly by the sun's heat, redistribution of the crust by water necessitates a sun heat received upon the earth's surface sufficient at least to maintain the temperature above that of permanent freezing. Newcomb* remarks:

> "If we reflect that a diminution of the solar heat by less than one-fourth its amount would probably mean an earth so cold that all the water on its surface would freeze, while an increase of much more than one-half would probably boil all the water away, it must be admitted that the balance of cause which would result in the sun radiating heat just fast enough to preserve the earth in its present state has probably not existed more than 10,000,000 years."

All we know of the earlier strata indicates a water mechanism for the denudation, comminution and deposition of rock. Exactly the division of this work between tidal and river

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* Popular Astronomy, p. 511.

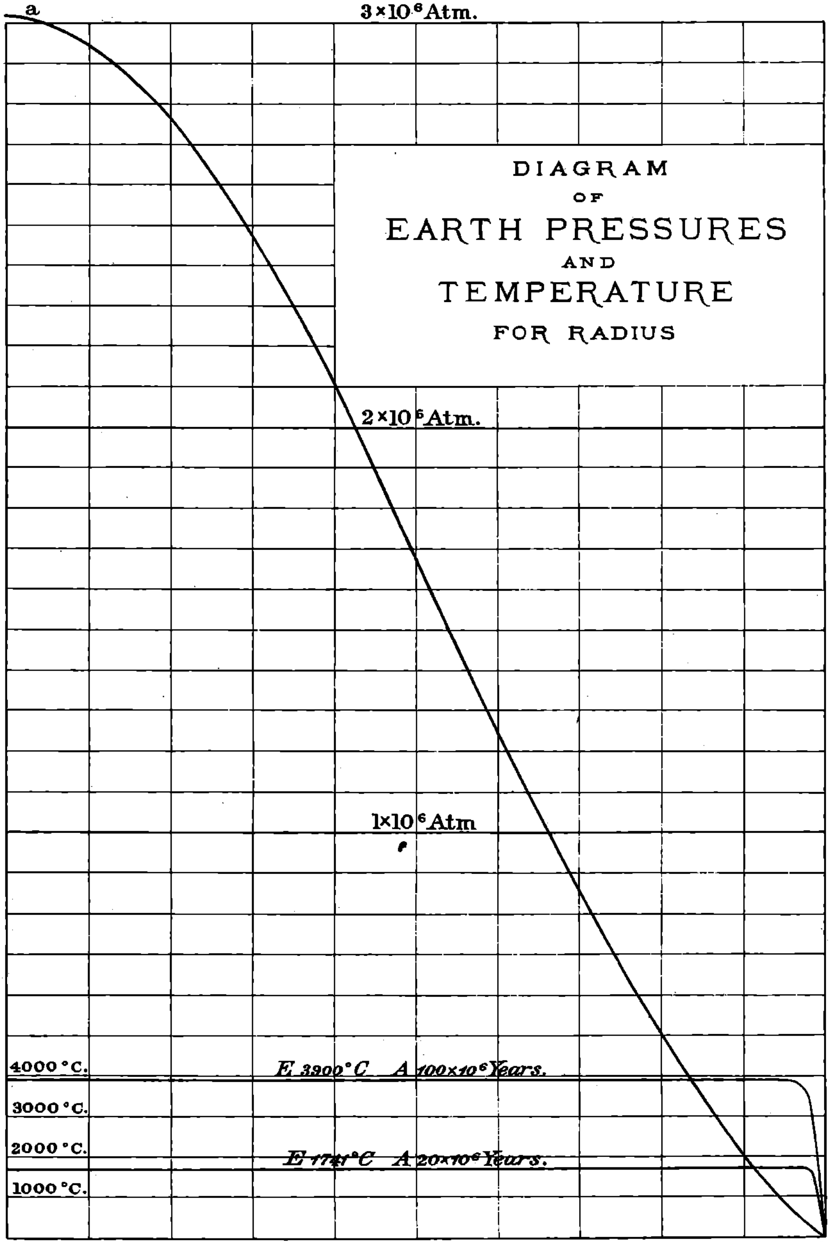
forces we may never know, but all evidences confirm the conviction that life was continuous from its earliest, or at least an early, appearance and hence climate must have been continuously suitable for the circulation of continental waters. The range of temperature for the time since the beginning of the Huronian must have been well within Newcomb's limits. So that unless the selective absorption of either the sun's atmosphere or the earth's or both have varied reciprocally or concurrently with radiation, solar emission cannot have had a wide range of either secular or paroxysmal change.

Nevertheless the age assigned to the sun by Helmholtz and Kelvin (15x10^6 or 20x10^6 years) communicated a shock from which geologists have never recovered. The thermodynamic reasoning on which the brevity of the sun's life is reached stands undisturbed, yet so powerful is the influence of the old uniformation method of estimating the age of the total stratified crust, that to many geologists it has seemed easier to reject the physical conclusions than to seek a source of error in our own very vulnerable methods.

If as I hold, Kelvin's suggestions as to ellipticity and tidal retardation do not apply to an earth readily deformable by slow stress as this one evidently is, there remain but three earth-ages to be weighed—Kelvin's value from terrestrial refrigeration which this paper seeks to advance to a new precision, Helmholtz and Kelvin's age of the sun which must sharply limit the date of the redistributed earth crust, and the old stratigraphical method. From this point of view the conclusions of the earlier part of this paper become of interest. The earth's age, about twenty-four millions of years, accords with the fifteen or twenty millions found for the sun.

In so far as future investigation shall prove a secular augmentation of the sun's emission from early to present time in conformity with Lane's law, his age may be lengthened, and further study of terrestrial conductivity will probably extend that of the earth.

Yet the concordance of results between the ages of sun and earth, certainly strengthens the physical case and throws the burden of proof upon those who hold to the vaguely vast age, derived from sedimentary geology.


a
3×10⁶ Atm.
DIAGRAM
OF
EARTH PRESSURES
AND
TEMPERATURE
FOR RADIUS
2×10⁶ Atm.
1×10⁶ Atm
4000 °C.
E 3900° C A 100×10⁶ Years.
3000 °C.
2000 °C.
E 1741°C A 20×10⁶ Years.
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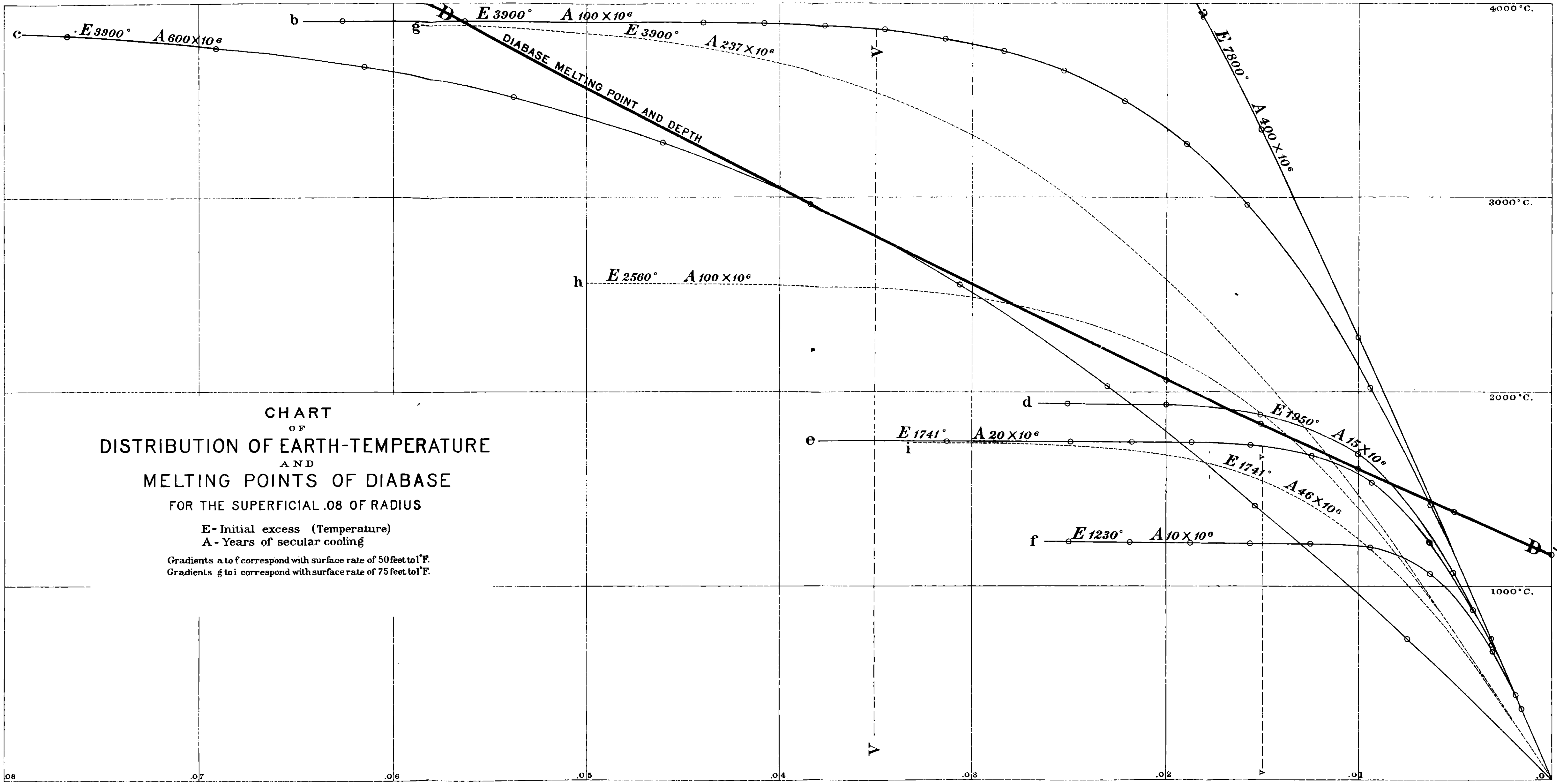

CHART OF DISTRIBUTION OF EARTH-TEMPERATURE AND MELTING POINTS OF DIABASE FOR THE SUPERFICIAL .08 OF RADIUS

E - Initial excess (Temperature)
A - Years of secular cooling

Gradients a to f correspond with surface rate of 50 feet to 1°F.
Gradients g to i correspond with surface rate of 75 feet to 1°F.