



Dynamic transition between fixed- and mobile-bed: mathematical and numerical aspects

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Free-surface flows with high sediment transport (as debris flow or hyper-concentrated flow) are composed by a mixture of fluid and solid phase, usually water and sediment. When these flows propagate over loose beds, particles constituting the mixture of water and sediments strongly interact with the ones forming the bed, leading to erosion or deposition. However, there are lots of other situations when the mixture flows over rigid bedrocks or over artificially paved transects, so there is no mass exchange between bed and mixture. The two situations are usually referred to as, respectively, mobile- and fixed-bed conditions.

From a mathematical point of view, the systems of Partial Differential Equations (PDEs) that describe these flows derive from mass and momentum balance of both phases, but, the two resulting PDEs systems are different. The main difference concerns the concentration: in the mobile-bed condition, the concentration is linked to the local flow conditions by means of a suitable rheological relation, while in the fixed-bed case, the concentration is an unknown of the problem.

It is quite common that a free surface flow with high sediment transport, in its path, encounters both conditions. In the recent work of Rosatti & Zugliani 2015, the mathematical and numerical description of the transition between fixed- and mobile-bed was successfully resolved, for the case of low sediment transport phenomena, by the introduction of a suitable erodibility variable and satisfactory results were obtained. The main disadvantage of the approach is related to the erodibility variable, that changes in space, based on bed characteristics, but remains constant in time. However, the nature of the bed can change dynamically as result of deposition over fixed bed or high erosion over mobile bed.

With this work, we extend the applicability of the mentioned approach to the more complex PDEs describing the hyper-concentrated flow. Moreover, we introduce a strategy that allows a dynamic time variation of the erodibility variable. The issue of the dynamic transition between fixed- and mobile-bed condition is tackled, from a numerical point of view, using a particular predictor corrector technique that compare the transported concentration related with the fixed bed and the equilibrium concentration, deriving from a closure relation, associated to the mobile bed condition.

Through a comparison between exact solution, built using the generalized Rankine – Hugoniot condition, and the numeric results, we highlight capabilities and limits of this enhanced technique.

Bibliography:

G. Rosatti and D. Zugliani, 2015. “Modelling the transition between fixed and mobile bed conditions in two-phase free-surface flows: The Composite Riemann Problem and its numerical solution”. *Journal of Computational Physics*, 285:226–250