



## Numerical Inversion with Full Estimation of Variance-Covariance Matrix

Vasso Saltogianni and Stathis Stiros  
University of Patras, Patras, Greece

Various geophysical problems are described by redundant systems of highly non-linear systems of equations with  $\geq 3$  unknown variables. Such systems are not possible to be solved with formal algebraic techniques, and are usually solved using sampling methods (mostly Monte Carlo-based), gradual optimization of certain of the unknown variables, a priori fixing of the values of some variables or in the vicinity of approximate solutions. In many cases, especially in the modeling of activated faults or of magma sources from surface displacements, such methods lead to sub-optimal solutions (trapped in local extrema, high uncertainties (trade-off) between certain variables, etc.) and highly influence the understanding/ modeling of certain complex geophysical processes.

In order to overcome these difficulties we proposed an alternative, topology-based, deterministic, numerical approach for the inversion of such systems of equations with  $n$  unknown variables, the TOPological INVersion (TOPINV) algorithm. TOPINV has been inspired from traditional positioning and the geodetic theory and is based on the intersection of spaces and the identification of clusters of points which satisfy observations equations. It is not based on the minimization of a certain cost function and involves only forward computations, hence avoids computational errors.

The basic concept is to assume discrete possible ranges of each variable, and from these ranges to define a grid  $G$  in  $R^n$  space containing the true solution (discrete search hyperspace). Each point of this hyper-grid is then tested whether it satisfies or not the observations, given their uncertainty level. This is possible by transforming equations to double (absolute value) inequalities using a single optimization parameter and a trial-and-error approach. The optimal (minimal) space containing one or more solutions in the form of one or more compact clouds (sets) of gridpoints satisfying the system of equations is then selected, and single-point, stochastic optimal solutions are computed as the center of gravity of these sets. A full Variance-Covariance Matrix (VCM) of each solution can be directly computed as second statistical moment.

The overall method and the software have been tested with synthetic data (accuracy-oriented approach) in the modeling of magma chambers in the Santorini volcano and the modeling of double-fault earthquakes, i.e. to inversion problems with up to 18 unknowns.