



## **Generalized correlation integral vectors: A distance concept for chaotic dynamical systems**

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It is difficult to distinguish systematic trends from natural variability of data in many important applications, such as weather or climate systems. The practical challenge of estimating parameters in chaotic systems is related to the fact that a fixed model parameter does not correspond to a unique model integration, but to a set of quite different solutions as obtained for example by setting slightly different initial values. But while all such trajectories are different, they approximate the same underlying attractor and should be considered in this sense equivalent. In this paper, we propose a statistical approach to quantify such “sameness” of trajectories, and to distinguish trajectories that are significantly different. Various formulations of fractal dimensions have been developed to characterize the geometry of such attractors. The aim of this paper is to modify one of these, the so-called correlation dimension, to develop a way to quantify the variability of samples of an attractor by mapping the respective phase space trajectories onto vectors, whose statistical distribution can be empirically estimated. The distributions turn out to be Gaussian, which provide us a well-defined statistical tool to compare the trajectories. We use the approach for the task of parameter estimation of chaotic systems. The methodology is illustrated using computational examples for both low and high dimensional systems, together with a framework for Markov chain Monte Carlo sampling to produce posterior distributions of model parameters.