



Fractal differentiation and integration and implication on singularity analysis of extreme geodynamics

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Singularity theory states that extreme geo-processes result in anomalous amounts of energy release or material accumulation within a narrow spatial–temporal interval. The products (e.g. mass density and energy density) caused by extreme geo-processes depict singularity without the ordinary derivative and antiderivative (integration) properties. Based on the definition of fractal density, the density measured in fractal dimensional space, in the current paper the author is proposing several operations including fractal derivative and fractal integral to analyze singularity of fractal density. While the ordinary derivative including fractional derivatives as a fundamental tool measuring the sensitivity of change of function (quantity as dependent variable) with change of another quantity as independent variable, the changes are measured in the ordinary space with additive property, fractal derivative (antiderivative) measures the ratio of changes of two quantities measured in fractal space–fractal dimensional space. For example, if the limit of ratio of increment of quantity (Δf) over the associated increment of time (Δt^α) measured in α – dimensional space approaches to a finite value, then the limit is referred a α -dimensional fractal derivative of function f and denoted as

$$f'_\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta f}{\Delta t^\alpha} = \frac{df}{dt^\alpha}$$

According to the definition of the fractal derivative the ordinary derivative becomes the special case if the space becomes non-fractal space with α value as an integer. In the rest of the paper we demonstrate that fractal density concept and fractal derivative can be applied in describing singularity property of products caused by extreme or avalanche events. The extreme earth-thermal processes such as hydrothermal mineralization occurred in the earth crust, heat flow over ocean ridges, igneous activities or juvenile crust grows, originated from cascade earth dynamics (mantle convection, plate tectonics, and continent crust grow etc.) were analyzed by fractal derivative method according to fractal density of mass accumulation or energy release.