Enhanced micropolar model for wave propagation in granular materials

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In the description of material elastic behavior, the classical theory of elasticity consists of a macroscopic material description. The material is not described at the micro-level by considering the displacement of the different particles in interaction, but is described as a continuum by considering macroscopic quantities as stress and strain. The classical elasticity theory can be viewed as first gradient of the displacement field approximation of the solid state theory and is valid in the long wavelength limit. Granular media, due to their micro-inhomogeneous character, are not well described by the standard continuum theory of elasticity. By contrast to classical continua where the sizes of the vibrating particles are assumed to be negligible compared to the distance between the particles, the sizes of the particles in a granular assembly are comparable to the distance between neighbors. In addition, considering the sliding, torsion and rolling resistances at the level of the contacts between the particles, a consistent description of the elasticity of a granular medium needs to take into account the rotational degrees of freedom of each individual particle.

The elastic behavior of crystalline structures of monodisperse beads can be efficiently described by a discrete model, where the displacement and rotation of each individual bead are taken into account. Nevertheless, the discrete model can be solved analytically only for well-know regular crystalline structure, the case of a random assembly of beads is too complex for large systems. A continuum formulation is more suitable for random assemblies of beads different from the ideal crystalline case. The generalization of the classical elasticity theory accounting for the rotational degrees of freedom of point bodies is known as the Cosserat or micropolar theory.

In this work, the vibration properties of a face-centered cubic structure of monodisperse granular crystal are predicted using a discrete model as well as a Cosserat model. The Cosserat model is derived from the discrete model through an expansion of the discrete displacement and particle rotation to continuum field variables. The long wavelength approximation of these two models are compared and, considering the discrete model as the reference, the Cosserat model shows inconsistent predictions of the bulk wave dispersion relations. The discrepancies between the two models are explained by an insufficient modeling of one of the particle interactions in the Cosserat model. A enhanced micropolar model is proposed to correctly describe all the particle interactions by including a new elastic tensor from the second order gradient micropolar theory. The enhanced micropolar model involve the minimum number of elastic constant to consistently predicts the bulk mode dispersion relations in the long wavelength limit.