

A tentative of interpreting Richards' Equation in media with high heterogeneity by Filippov theory

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The numerical solution of Richards' equation is accomplished by means of method of lines, that typically allows the spatial derivative to be approximated by some finite element scheme, in such a way that any solver for ODEs can be used. The ψ -based form is used, i.e.

$$C(\psi) \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z} \left[K(\psi) \left(\frac{\partial \psi}{\partial z} - 1 \right) \right], \quad (1)$$

for suitable choices of hydraulic capacity function C and hydraulic conductivity function K .

The real challenge is modelling the infiltration at the interface between two media with high heterogeneity. The interface between two layered media with very different characteristics can be handled as a discontinuity surface. The effort is to review this case as a differential system with discontinuous right-hand side and to clarify the meaning of *crossing* and *sliding* in this context, according to Filippov theory. For our scopes, the temporal derivative has been approximated by means of a finite difference method in such a way that the numerical integration is accomplished with respect to the spatial variable z in (1): this choice is particularly convenient since it allows to have a Filippov system, with a state-dependent threshold, and to have a possible sliding behavior.