Geophysical Research Abstracts
Vol. 18, EGU2016-16536, 2016
EGU General Assembly 2016

2D instabilities of surface gravity waves on a linear shear current

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Periodic 2D surface water waves propagating steadily on a rotational current have been studied by many authors (see [1] and references therein). Although the recent important theoretical developments have confirmed that periodic waves can exist over flows with arbitrary vorticity, their stability and their nonlinear evolution have not been much studied extensively so far. In fact, even in the rather simple case of uniform vorticity (linear shear), few papers have been published on the effect of a vertical shear current on the side-band instability of a uniform wave train over finite depth. In most of these studies [2-5], asymptotic expansions and multiple scales method have been used to obtain envelope evolution equations, which allow eventually to formulate a condition of (linear) instability to long modulational perturbations. It is noted here that this instability is often referred in the literature as the Benjamin-Feir or modulational instability.

In the present study, we consider the linear stability of finite amplitude two-dimensional, periodic water waves propagating steadily on the free surface of a fluid with constant vorticity and finite depth. First, the steadily propagating surface waves are computed with steepness up to very close to the highest, using a Fourier series expansions and a collocation method, which constitutes a simple extension of Fenton's method [6] to the cases with a linear shear current. Then, the linear stability of these permanent waves to infinitesimal 2D perturbations is developed from the fully nonlinear equations in the framework of normal modes analysis. This linear stability analysis is an extension of [7] to the case of waves in the presence of a linear shear current and permits the determination of the dominant instability as a function of depth and vorticity for a given steepness. The numerical results are used to assess the accuracy of the vor-NLS equation derived in [5] for the characteristics of modulational instabilities due to resonant four-wave interactions, as well as to study the influence of vorticity and nonlinearity on the characteristics of linear instabilities due to resonant five-wave and six-wave interactions. Depending on the dimensionless depth, superharmonic instabilities due to five-wave interactions can become dominant with increasing positive vorticiy.

Acknowledgments: This work was supported by the Direction Générale de l'Armement and funded by the ANR project n°. ANR-13-ASTR-0007.

References

[1] A. Constantin, Two-dimensionality of gravity water flows of constant non-zero vorticity beneath a surface wave train, Eur. J. Mech. B/Fluids, 2011, 30, 12–16.

[2] R. S. Johnson, On the modulation of water waves on shear flows, Proc. Royal Soc. Lond. A., 1976, 347, 537–546.

[3] M. Oikawa, K. Chow, D. J. Benney, The propagation of nonlinear wave packets in a shear flow with a free surface, Stud. Appl. Math., 1987, 76, 69–92.

[4] A. I Baumstein, Modulation of gravity waves with shear in water, Stud. Appl. Math., 1998, 100, 365–90.

[5] R. Thomas, C. Kharif, M. Manna, A nonlinear Schrödinger equation for water waves on finite depth with constant vorticity, Phys. Fluids, 2012, 24, 127102.

[6] M. M Rienecker, J. D Fenton, A Fourier approximation method for steady water waves , J. Fluid Mech., 1981, 104, 119–137

[7] M. Francius, C. Kharif, Three–dimensional instabilities of periodic gravity waves in shallow water, J. Fluid Mech., 2006, 561, 417–437