



Explicit wave action conservation for water waves on vertically sheared flows

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Water waves almost always propagate on currents with a vertical structure such as currents directed towards the beach accompanied by an under-current directed back toward the deep sea or wind-induced currents which change magnitude with depth due to viscosity effects. On larger scales they also change their direction due to the Coriolis force as described by the Ekman spiral. This implies that the existing wave models, which assume vertically-averaged currents, is an approximation which is far from realistic. In recent years, ocean circulation models have significantly improved with the capability to model vertically-sheared current profiles in contrast with the earlier vertically-averaged current profiles. Further advancements have coupled wave action models to circulation models to relate the mutual effects between the two types of motion. Restricting wave models to vertically-averaged non-turbulent current profiles is obviously problematic in these cases and the primary goal of this work is to derive and examine a general wave action equation which accounts for these shortcomings.

The formulation of the wave action conservation equation is made explicit by following the work of Voronovich (1976) and using known asymptotic solutions of the boundary value problem which exploit the smallness of the current magnitude compared to the wave phase velocity and/or its vertical shear and curvature. The adopted approximations are shown to be sufficient for most of the conceivable applications. This provides correction terms to the group velocity and wave action definition accounting for the shear effects, which are fitting for application to operational wave models.

In the limit of vanishing current shear, the new formulation reduces to the commonly used Bretherton & Garrett (1968) no-shear wave action equation where the invariant is calculated with the current magnitude taken at the free surface. It is shown that in realistic oceanic conditions, the neglect of the vertical structure of the currents in wave modelling which is currently universal, might lead to significant errors in wave amplitude and the predicted wave ray paths.

An extension of the work toward the more complex case of turbulent currents will also be discussed.