



Fourier spectrum and phases for a signal in a finite interval

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When investigating the physics of turbulent media, as the solar wind or the magnetosheath plasmas, obtaining accurate Fourier spectra and phases is a crucial issue. For the different fields, the spectra allow in particular verifying whether one or several power laws can be determined in different frequency ranges. Accurate phases are necessary as well for all the "higher order statistics" studies in Fourier space, the coherence ones and for the polarization studies.

Unfortunately, the Fourier analysis is not unique for a finite time interval of duration T : the frequencies lower than $1/T$ have a large influence on the result, which can hardly be controlled. This unknown "trend" has in particular the effect of introducing jumps at the edges of the interval, for the function under study itself, as well as for all its derivatives. The Fourier transform obtained directly by FFT (Fast Fourier Transform) is generally much influenced by these effects and cannot be used without care for wide band signals.

The interference between the jumps and the signal itself provide in particular characteristic "hairs" on the spectrum, which are clearly visible on it with $df \approx 1/T$. These fluctuations are usually eliminated by smoothing the spectrum, or by averaging several successive spectra. Nevertheless, such treatments introduce uncertainties on the spectral laws (the phases being anyway completely lost). Windowing is also a method currently used to suppress or decrease the jumps, but it modifies the signal (the windowed trend has a spectrum, which is convolved with the searched one) and the phases are generally much altered.

Here, we present a new data processing technique to circumvent these difficulties. It takes advantage of the fact that the signal is generally not unknown out of the interval under study: the complete signal is tapered to this interval of interest thanks to a new window function, sharp but not square. This window function is chosen such that the spectrum obtained can be deconvolved almost exactly, through a minimization procedure based on the "weak-hypothesis" that it is smooth at the scale of a few successive spectral points. Then, a later step allows reconstructing the phases. Tests with synthetic data and first applications to Cluster data are presented, which demonstrate the capability of the method to better estimate the Fourier spectra.