Geophysical Research Abstracts
Vol. 17, EGU2015-5308, 2015
EGU General Assembly 2015

New Scaling Model for Variables and Increments with Heavy-Tailed Distributions

Monica Riva (1,2), Alberto Guadagnini (1,2), and Shlomo P. Neuman (2)

(1) Politecnico di Milano, Dip. di Ingegneria Civile e Ambientale, Milano, Italy, (2) Department of Hydrology and Water Resources, University of Arizona, Tucson, Arizona 85721, USA

Many earth, environmental, ecological, biological, physical, social, financial and other variables, Y, exhibit (i) asymmetry in sample frequency distributions, as well as (*ii*) symmetry in distributions of their spatial and/or temporal increments, ΔY, at diverse separation distances (or lags), with sharp peaks and heavy tails which appear to decay asymptotically (often toward exponential tails of the Gaussian distribution) as lag increases. No model known to us captures all of these behaviors in a unique and consistent manner. We propose a new model that does so upon treating $Y(x)$ as a random function of a coordinate x in the Euclidean (spatial) domain or time, forming a stationary random field (or process) with constant ensemble mean (expectation) $\langle Y \rangle$. We express the zero-mean random fluctuation $Y'(x) = Y(x) - \langle Y \rangle$ in sub-Gaussian form. In the classical sub-Gaussian form, $Y'(x) = UG(x)$, where $G(x)$ is a zero-mean stationary Gaussian random field (or process) and the subordinator U is an independent non-negative random variable (Samorodnitsky and Taqqu, 2014). We generalize this by writing $Y'(x) = U(x)G(x)$ in which $U(x)$ is *iid*. This enables us to analyze, and synthetically generate, heavy-tailed non-Gaussian distributions of both Y and ΔY in both probability space (across an infinite ensemble of random realizations) and real space (in a single realization). We derive analytical expressions for probability distribution functions (*pdf*s) of Y and ΔY as well as their lead statistical moments. We show that when U is lognormal Y follows the well-known normal-lognormal distribution (NLN, of which the Gaussian distribution is a particular case) with constant parameters. The NLN *pdf* has been successfully used to interpret financial (Clark, 1973) and environmental (Guadagnini et al., 2014) data. However, ΔY is not NLN and forcing the latter on the former gives a false and widely accepted impression that (a) parameters of the ΔY *pdf* vary with lag and (b) the *pdfs* of Y and ΔY are mutually inconsistent. Our new theory shows that both "observations" stem from a misconception. We eliminate these apparent inconsistencies by deriving analytically the *pdf* of ΔY from that of Y, given that the latter is NLN. In our model the tail of the ΔY *pdf* scales with the correlation coefficient of G and thereby with lag; all other parameters of the *pdf* remaining unaltered with lag. We illustrate key features of our new model by way of synthetic examples. We then propose and test a methodology to estimate, accurately and efficiently, all model parameters using sample moments of both Y and ΔY. We end by exemplifying the use of our new model and method of inference by applying them to neutron porosity data from deep boreholes.

References

Clark, P.K. (1973), A subordinated stochastic process model with finite variance for speculative prices, Econometrica, 41, 1.

Samorodnitsky, G., and M.S. Taqqu (1994), Stable Non-Gaussian Random Processes, Chapman and Hall, New York.

Guadagnini, A., S.P. Neuman, T. Nan, M. Riva, and C.L. Winter (2015), Scalable statistics of correlated random variables and extremes applied to deep borehole porosities, Hydrology and Earth System Sciences, *in press*.