



Consistent nonlinear deterministic and stochastic evolution equations for deep to shallow water wave shoaling

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One-dimensional deterministic and stochastic evolution equations are derived for the dispersive nonlinear waves while taking dissipation of energy into account. The deterministic nonlinear evolution equations are formulated using operational calculus by following the approach of Bredmose et al. (2005). Their formulation is extended to include the linear and nonlinear effects of wave dissipation due to friction and breaking. The resulting equation set describes the linear evolution of the velocity potential for each wave harmonic coupled by quadratic nonlinear terms. These terms describe the nonlinear interactions between triads of waves, which represent the leading-order nonlinear effects in the near-shore region. The equations are translated to the amplitudes of the surface elevation by using the approach of Agnon and Sheremet (1997) with the correction of Eldeberky and Madsen (1999).

The only current possibility for calculating the surface gravity wave field over large domains is by using stochastic wave evolution models. Hence, the above deterministic model is formulated as a stochastic one using the method of Agnon and Sheremet (1997) with two types of stochastic closure relations (Benney and Saffman's, 1966, and Holloway's, 1980). These formulations cannot be applied to the common wave forecasting models without further manipulation, as they include a non-local wave shoaling coefficients (i.e. ones that require integration along the wave rays). Therefore, a localization method was applied (see Stiassnie and Drimer, 2006, and Toledo and Agnon, 2012). This process essentially extracts the local terms that constitute the mean nonlinear energy transfer while discarding the remaining oscillatory terms, which transfer energy back and forth.

One of the main findings of this work is the understanding that the approximated non-local coefficients behave in two essentially different manners. In intermediate water depths these coefficients indeed consist of rapidly oscillating terms, but as the water depth becomes shallow they change to an exponential growth (or decay) behavior. Hence, the formerly used localization technique cannot be justified for the shallow water region. A new formulation is devised for the localization in shallow water, it approximates the nonlinear non-local shoaling coefficient in shallow water and matches it to the one fitting to the intermediate water region. This allows the model behavior to be consistent from deep water to intermediate depths and up to the shallow water regime. Various simulations of the model were performed for the cases of intermediate, and shallow water, overall the model was found to give good results in both shallow and intermediate water depths.

The essential difference between the shallow and intermediate nonlinear shoaling physics is explained via the dominating class III Bragg resonances phenomenon. By inspecting the resonance conditions and the nature of the dispersion relation, it is shown that unlike in the intermediate water regime, in shallow water depths the formation of resonant interactions is possible without taking into account bottom components.

References

- Agnon, Y. & Sheremet, A. 1997 Stochastic nonlinear shoaling of directional spectra. *J. Fluid Mech.* 345, 79-99.
Benney, D. J. & Saffman, P. G. 1966 Nonlinear interactions of random waves. *Proc. R. Soc. Lond. A* 289, 301-321.
Bredmose, H., Agnon, Y., Madsen, P.A. & Schaffer, H.A. 2005 Wave transformation models with exact second-order transfer. *European J. of Mech. - B/Fluids* 24 (6), 659-682.
Eldeberky, Y. & Madsen, P. A. 1999 Deterministic and stochastic evolution equations for fully dispersive and weakly nonlinear waves. *Coastal Engineering* 38, 1-24.
Kaihatu, J. M. & Kirby, J. T. 1995 Nonlinear transformation of waves in infinite water depth. *Phys. Fluids* 8, 175-188.
Holloway, G. 1980 Oceanic internal waves are not weak waves. *J. Phys. Oceanogr.* 10, 906-914.
Stiassnie, M. & Drimer, N. 2006 Prediction of long forcing waves for harbor agitation studies. *J. of waterways,*

port, coastal and ocean engineering 132(3), 166-171.

Toledo, Y. & Agnon, Y. 2012 Stochastic evolution equations with localized nonlinear shoaling coefficients. European J. of Mech. - B/Fluids 34, 13-18.