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A wave action equation for water waves propagating on vertically sheared flows

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The coexistence of motions of different scales in oceans and other natural water basins presents a challenge for their dynamic modeling. For water waves on currents, an asymptotic procedure exploiting the separation of scales allows the modeling of two motions of a qualitatively different nature, the fast shortwaves on the surface and the dynamics of the slow, long currents.

Most wave forecast models are based on the wave action equation which is a conservation equation which takes into account the propagation of the wave energy in geographic space, shoaling, refraction, diffraction and also source terms which account for generation, wave-wave interactions and dissipation of the energy.

Water waves almost always propagate on currents with a vertical structure such as currents directed towards the beach accompanied by an under-current directed back toward the deep sea or wind-induced currents which change magnitude with depth due to viscosity effects. On larger scales they also change their direction due to the Coriolis force as described by the Ekman spiral. This implies that the existing wave models, which assume vertically-averaged currents, is an approximation which is far from realistic.

In recent years, ocean circulation models have significantly improved with the capability to model vertically-sheared current profiles in contrast with the earlier vertically-averaged current profiles. Further advancements have coupled wave action models to circulation models to relate the mutual effects between the two types of motion. Restricting wave models to vertically-averaged current profiles is obviously problematic in these cases and the primary goal of this work is to derive and examine a general wave action equation which accounts for this shortcoming.

Combining two previous theoretical approaches [Voronovich, 1976; Skop, 1987], the developed wave action formulation greatly improves the representation of linear wave-current interaction in the case of tidal inlets, wind-induced currents, storm surges and undertow currents. In contrast to the case of vertically averaged ambient currents, the structure of the oscillatory flow under the wave depends on the current's vertical structure. Locally, this structure relates to the solution of the Rayleigh equation with appropriate surface and bottom boundary conditions, an essential step for creating an applicable explicit wave action formulation. For an arbitrary current profile the Rayleigh equation boundary-value problem does not have an exact analytical solution asymptotic solutions must be employed.

Two approximations are made to the new formulation; one assuming a small current, gradient and curvature, the Skop [1987] approximation, the other assuming no shear. The accuracy of these approximations are analysed numerically to determine the improvement of the shear to no shear approximations over the exact solutions to the group velocity Cg and the invariant Ivs. A two-layer profile and also a countercurrent vertical velocity profile were examined to determine the range of validity of the approximations for simple wave-current interaction problems.

Under the assumptions of a small current, small current gradient and small curvature, to the leading order, the new wave action formulation gives the same results as the no-shear formulation which is based on the famous work of Bretherton and Garrett [1968] and that the existing models need alter only their value of the current to be taken at the surface, rather than a depth-averaged value.

References

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