



Steady rotational water wave of finite amplitude on a linear shear current

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Considering an incompressible, inviscid fluid the equations of motion for water waves on a linear shear current reduce to a Poisson equation for the stream-function. This is a boundary value problem with the Bernoulli equation on the free surface and a 'no normal flow' condition on the flat bottom. Using a transformation developed by Dubreil-Jacotin the linear Poisson equation takes on a non-linear elliptic form. This is to be solved on a rectangular domain (x, ψ) which is 2π periodic in x . Vertically the domain extends from $\psi = 0$ on the free-surface to the value of the stream-function on the bottom (ψ_B). The boundary conditions are also transformed and the system is closed in such a way that the phase velocity, depth and Bernoulli constant are unknowns. The equations are discretised by means of a differentiation matrix method and solved using a simple Newton-Raphson solver in MATLAB. The result is a set of values for a variable $Z = z$ which determines the free-surface position and the entire rotational flow-field after reverse transformation. This paper will cover the analysis building up to initial, first order linear solutions of the elliptic problem as well as the numerical methods used to obtain fully non-linear basic flows. These will then be used in a stability analysis and will be extended by considering arbitrary vorticity distributions.