



## **Adaptively Forward Modelling the Spatial Magnetic Effects Due to a Magnetized Structure by Tesseroids in Spherical Coordinate System**

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The continually accumulated magnetic measurements and also the reliable global lithospheric magnetic anomaly field models obtained by CHAMP satellite and Swarm constellation of three satellites, now present a requirement and also a challenge to develop the realistic forward modeling methods for the magnetic effects (i.e. magnetic potential, vector and gradient tensor) that take into account the curvature of the Earth. The spatial discretization by a series of elementary tesseroids (spherical prisms, SPs) is utilized to approximate the complex magnetized source by the principle of superposition and saturate the source volume without “holes”.

Since there is no analytic solution for the magnetic effects of the SP, we explicitly present three kinds of efficient forward modeling methods for approximate calculation using Taylor's series expansion (TSE) to fourth-order, Gauss-Legendre quadrature integration (GLQI) and approximations by Cartesian elements including the magnetic dipole (MD) and rectangular prism (RP). Our derived new formulas do not suffer from the polar singularity and using the approximate approaches and subdivision technique, therefore, can be employed for any computing point with a required level of accuracy on the globe. Both theoretical analysis and numerical investigations suggest that the accuracy of modeling by the SP is significantly dependent on its geometric shape (i.e. size, latitude and depth) and particularly the distance between the source and the observation (DSO for short). Accuracies of forward modeling by all methods are relatively worse near the source but better far away the source. Besides, the numerical analysis shows that the error of magnetic potential is lower than those of magnetic vector and gradient tensor, and that of the gradient tensor is the highest but the error's decay of the tensor is the fastest. Analysis of accuracy shows that MD method is equivalent to GLQI when node is zero, and TSE method is nearly equivalent to GLQI when nodes are ones. The RP method shows a better accuracy when the computing point nears the source located in the low latitude but worse in the high latitude where the RP fails to depict the actual geometric shape of the tesseroid. Because the error decays most quickly as the distance between the source and the computing point increasing, the 3D GLQI with higher than first-order Gaussian nodes is most efficiently combined with the reasonable subdivision technique. Investigation of computational efficiency indicates that, in practical applications, combination of different methods and reasonable choices for the number of subdivision and the Gaussian nodes of the GLQI can produce the most accurate results in an acceptable computational time. The calculated magnetic effects of the global lithospheric magnetization model show that there are little differences between the results at the satellite altitude by the MD method and by the 3D GLQI method but very large differences at the low altitude, which means that the MD method will introduce large errors and become worse and worse as the altitude decreasing.

However, the corresponding computation is very time-consuming, especially when the geometric size of the source is large or the DSO is short. Because the error decays most quickly as the DSO increasing, the 3D GLQI with higher than first-order Gaussian nodes is most efficiently combined with the reasonable subdivision technique. Therefore, in order to reduce the computing time while maintaining the desired accuracy, an adaptive forward modeling scheme that consists of using a fixed Gaussian order and recursively subdivision is employed. As a key point in this approach, the relationships between the accuracy of the magnetic effects and the size-to-DSO ratio are focused and investigated. Numerical test shows that this adaptive approach improves the computational efficiency significantly.