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A simple probabilistic model of initiation of motion of poorly-sorted granular mixtures subjected to a turbulent flow

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Initiation of sediment motion is a classic problem of sediment and fluid mechanics that has been studied at wide range of scales. By analysis at channel scale one means the investigation of a reach of a stream, sufficiently large to encompass a large number of sediment grains but sufficiently small not to experience important variations in key hydrodynamic variables. At this scale, and for poorly-sorted hydraulically rough granular beds, existing studies show a wide variation of the value of the critical Shields parameter. Such uncertainty constitutes a problem for engineering studies. To go beyond Shields paradigm for the study of incipient motion at channel scale this problem can be can be cast in probabilistic terms.

An empirical probability of entrainment, which will naturally account for size-selective transport, can be calculated at the scale of the bed reach, using a) the probability density functions (PDFs) of the flow velocities $f_u(u|x_n)$ over the bed reach, where u is the flow velocity and x_n is the location, b) the PDF of the variability of competent velocities for the entrainment of individual particles, $f_{u_p}(u_p)$, where u_p is the competent velocity, and c) the concept of joint probability of entrainment and grain size. One must first divide the mixture in into several classes M and assign a correspondent frequency p_M. For each class, a conditional PDF of the competent velocity $f_{u_p}(u_p|M)$ is obtained, from the PDFs of the parameters that intervene in the model for the entrainment of a single particle:

$$\frac{u_p}{\sqrt{g(s-1)d_i}} = \Phi_u\left(\{C_k\}, \{\phi_k\}, \psi, \frac{u_p d_i}{\nu^{(w)}}\right)$$

where $\{C_k\}$ is a set of shape parameters that characterize the non-sphericity of the grain, $\{\phi_k\}$ is a set of angles that describe the orientation of particle axes and its positioning relatively to its neighbours, ψ is the skin friction angle of the particles, $u_p d_i/\nu^{(w)}$ is a particle Reynolds number, d_i is the sieving diameter of the particle, g is the acceleration of gravity and Φ_u is a general function. For the same class, the probability density function of the instantaneous turbulent velocities $f_u(u|M)$ can be obtained from judicious laboratory or field work. From these probability densities, the empirical conditional probability of entrainment of class M is

$$P(E|M) = \int_{-\infty}^{+\infty} P(u > u_p|M)\; f_{u_p}(u_p|M) du_p$$

where $P(u > u_p|M) = \int_{u_p}^{+\infty} f_u(u|M)du$. Employing a frequentist interpretation of probability, in an actual bed reach subjected to a succession of N (turbulent) flows, the above equation states that the fraction $N\,P(E|M)$ is the number of flows in which the grains of class M are entrained. The joint probability of entrainment and class M is given by the product $P(E|M)p_M$. Hence, the channel scale empirical probability of entrainment is the marginal probability

$$P(E) = \sum_M P(E|M)p_M$$

since the classes M are mutually exclusive.

Fractional bedload transport rates can be obtained from the probability of entrainment through

$$q_{s_M} = E_M \ell_{s_M}$$

where q_{s_M} is the bedload discharge in volume per unit width of size fraction M, E_M is the entrainment rate per unit bed area of that size fraction, calculated from the probability of entrainment as $E_M = P(E|M)p_M(1-\lambda)d/(2T)$ where d is a characteristic diameter of grains on the bed surface, λ is the bed porosity, T is the integral length scale of the longitudinal velocity at the elevation of crests of the roughness elements and ℓ_{s_M} is the mean displacement length of class M.

Fractional transport rates were computed and compared with experimental data, determined from bedload samples collected in a 12 m long 40 cm wide channel under uniform flow conditions and sediment recirculation. The median diameter of the bulk bed mixture was 3.2 mm and the geometric standard deviation was 1.7. Shields parameters ranged from 0.027 and 0.067 while the boundary Reynolds number ranged between 220 and 376. Instantaneous velocities were measured with 2-component Laser Doppler Anemometry.

The results of the probabilist model exhibit a general good agreement with the laboratory data. However the probability of entrainment of the smallest size fractions is systematically underestimated. This may be caused by phenomena that is absent from the model, for instance the increased magnitude of hydrodynamic actions following the displacement of a larger sheltering grain and the fact that the collective entrainment of smaller grains following one large turbulent event is not accounted for.

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