



Comparison of various multifractal approaches to analyze the intermittent magnetic fluctuations observed in the Earth's magnetospheric cusp

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Several approaches exist to compute the multifractal characteristics of an intermittent set of fluctuations. First, the classical method based on the computation of the partition function uses the full set of fluctuations. Since it is dominated by the more numerous fluctuations of small amplitudes, this method can mask the fractal characteristics of minor fluctuations of much larger amplitude. To solve this issue, a new method was developed by Chang & Wu (2008) : the Rank-Ordered Multifractal Analysis (ROMA) The ROMA method offers a natural connection between the one-parameter monofractal scaling idea and the multifractal phenomenon of intermittency. The key-element in ROMA is to find $s(Y)$, the spectrum of the scaling exponents, and $Ps(Y)$, the scaled Probability Distribution Function (PDFs), from the raw PDFs of the variable X at various scales τ , $P(X,\tau)$, with the following scaling:

$$P(X,\tau) \tau^{s(Y)} = Ps(Y) \text{ with } Y = X/\tau^{s(Y)}$$

The first (direct) method is to use range-limited structure functions in a sufficiently small range of the scaled variable Y and search for the value of monofractal exponent $s(Y)$. A drawback of this approach is that the range of Y must be large enough to ensure that the statistics is meaningful. As a consequence, some cross-over behavior between fluctuations with different monofractal exponents can lead to an ambiguity with several solutions $s(Y)$ for some ranges of Y . Also the multifractal spectrum produced is step-wise discontinuous. To overcome these difficulties, Wu & Chang (2011) have suggested a refined method where a value of the parameter s is assumed and the corresponding value of Y ensuring a collapse of the raw PDFs is searched for. The advantage of this latter approach is that $s(Y)$ and $Ps(Y)$ can be obtained for single values of Y . The two ROMA methods and the partition function method are used on a set of intermittent magnetic field fluctuations observed by the Cluster spacecraft in the Earth's magnetospheric cusp. Results are presented and discussed.

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