



The non-equilibrium statistical mechanics of a simple geophysical fluid dynamics model

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Lorenz [1] has devised a dynamical system that has proved to be very useful as a benchmark system in geophysical fluid dynamics. The system in its simplest form consists of a periodic array of variables that can be associated with an atmospheric field on a latitude circle. The system is driven by a constant forcing, is damped by linear friction and has a simple advection term that causes the model to behave chaotically if the forcing is large enough.

Our aim is to predict the statistics of Lorenz' model on the basis of a given average value of its total energy - obtained from a numerical integration - and the assumption of statistical stationarity. Our method is the principle of maximum entropy [2] which in this case reads: the information entropy of the system's probability density function shall be maximal under the constraints of normalization, a given value of the average total energy and statistical stationarity. Statistical stationarity is incorporated approximately by using 'stationarity constraints', i.e. by requiring that the average first and possibly higher-order time-derivatives of the energy are zero in the maximization of entropy.

The analysis [3] reveals that, if the first stationarity constraint is used, the resulting probability density function rather accurately reproduces the statistics of the individual variables. If the second stationarity constraint is used as well, the correlations between the variables are also reproduced quite adequately. The method can be generalized straightforwardly and holds the promise of a viable non-equilibrium statistical mechanics of the forced-dissipative systems of geophysical fluid dynamics.

[1] E.N. Lorenz, 1996: Predictability - A problem partly solved, in Proc. Seminar on Predictability (ECMWF, Reading, Berkshire, UK), Vol. 1, pp. 1-18.

[2] E.T. Jaynes, 2003: Probability Theory - The Logic of Science (Cambridge University Press, Cambridge).

[3] W.T.M. Verkley and C.A. Severijns, 2014: The maximum entropy principle applied to a dynamical system proposed by Lorenz, Eur. Phys. J. B, 87:7, <http://dx.doi.org/10.1140/epjb/e2013-40681-2> (open access).