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# The forward and adjoint sensitivity methods of glacial isostatic adjustment: Existence, uniqueness and time-differencing scheme

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In this study, two new methods for computing the sensitivity of the glacial isostatic adjustment (GIA) forward solution with respect to the Earth's mantle viscosity are presented: the forward sensitivity method (FSM) and the adjoint sensitivity method (ASM). These advanced formal methods are based on the time-domain,spectral-finite element method for modelling the GIA response of laterally heterogeneous earth models developed by Martinec (2000). There are many similarities between the forward method and the FSM and ASM for a general physical system. However, in the case of GIA, there are also important differences between the forward and sensitivity methods. The analysis carried out in this study results in the following findings.

First, the forward method of GIA is unconditionally solvable, regardless of whether or not a combined ice and ocean-water load contains the first-degree spherical harmonics. This is also the case for the FSM, however, the ASM must in addition be supplemented by nine conditions on the misfit between the given GIA-related data and the forward model predictions to guarantee the existence of a solution. This constrains the definition of data least-squares misfit.

Second, the forward method of GIA implements an ocean load as a free boundary-value function over an ocean area with a free geometry. That is, an ocean load and the shape of ocean, the so-called ocean function, are being sought, in addition to deformation and gravity-increment fields, by solving the forward method. The FSM and ASM also apply the adjoint ocean load as a free boundary-value function, but instead over an ocean area with the fixed geometry given by the ocean function determined by the forward method. In other words, a boundary-value problem for the forward method of GIA is free with respect to determining (i) the boundary-value data over an ocean area and (ii) the ocean function itself, while the boundary-value problems for the FSM and ASM are free only with respect to (i).

Third, the forward method of GIA traditionally uses an explicit time-differencing scheme to calculate the evolution of the viscous stress tensor $\boldsymbol{\tau}$ over time. We show that this scheme is not sufficiently accurate for calculating the time evolution of the viscous gradient of the viscous stress tensor, $\nabla_{\vec{\eta}}\boldsymbol{\tau}$, underlying the FSM and ASM. This is particularly evident in the case where the size of time step is comparable to the shortest Maxwell relaxation time of the studied earth model, although the explicit time-differencing scheme of the forward method is still sufficiently accurate. The inaccuracy in $\nabla_{\vec{\eta}}\boldsymbol{\tau}$ which is unacceptable for computing the gradient of least-squares misfit, $\nabla_{\vec{\eta}}\,\chi^2$, can only be reduced by making the time steps shorter. This subsequently increases the demands on computation time and storage space, since the forward solution must be stored for solving the FSM and ASM. We propose and numerically test a new, semi-explicit time-differencing scheme for $\nabla_{\vec{\eta}}\boldsymbol{\tau}$ with a numerical accuracy comparable to the explicit scheme for $\boldsymbol{\tau}$.

Fourth, the FSM determines, among other forward sensitivities, the viscosity gradient of the ocean function which describes the sensitivity of coastal regions to a particular Earth's viscosity structure and shows how this sensitivity varies with time. This relation is particularly important for interpreting sea-level indicator data. The ASM does not, however, provide, this kind of sensitivity. On the other hand, the ASM is more efficient for the sensitivity analysis of models with a large number of viscosity parameters. In this sense, the FSM and ASM complement each other.