



Model calibration for ice sheets and glaciers dynamics: a general theory of inverse problems in glaciology

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Numerical modelling of the dynamic evolution of ice sheets and glaciers requires the solution of discrete equations which are based on physical principles (e.g. conservation of mass, linear momentum and energy) and phenomenological constitutive laws (e.g. Glen's and Fourier's laws). These equations must be accompanied by information on the forcing term and by initial and boundary conditions (IBC) on ice velocity, stress and temperature; on the other hand the constitutive laws involves many physical parameters, which possibly depend on the ice thermodynamical state.

The proper forecast of the dynamics of ice sheets and glaciers (forward problem, FP) requires a precise knowledge of several quantities which appear in the IBCs, in the forcing terms and in the phenomenological laws and which cannot be easily measured at the study scale in the field. Therefore these quantities can be obtained through model calibration, i.e. by the solution of an inverse problem (IP). Roughly speaking, the IP aims at finding the optimal values of the model parameters that yield the best agreement of the model output with the field observations and data. The practical application of IPs is usually formulated as a generalised least squares approach, which can be cast in the framework of Bayesian inference.

IPs are well developed in several areas of science and geophysics and several applications were proposed also in glaciology. The objective of this paper is to provide a further step towards a thorough and rigorous theoretical framework in cryospheric studies.

Although the IP is often claimed to be ill-posed, this is rigorously true for continuous domain models, whereas for numerical models, which require the solution of algebraic equations, the properties of the IP must be analysed with more care.

First of all, it is necessary to clarify the role of experimental and monitoring data to determine the calibration targets and the values of the parameters that can be considered to be fixed, whereas only the model output should depend on the subset of the parameters that can be identified with the calibration procedure and the solution to the IP. It is actually difficult to guarantee the existence and uniqueness of a solution to the IP for complex non-linear models. Also identifiability, a property related to the solution to the FP, and resolution should be carefully considered. Moreover, instability of the IP should not be confused with ill-conditioning and with the properties of the method applied to compute a solution. Finally, sensitivity analysis is of paramount importance to assess the reliability of the estimated parameters and of the model output, but it is often based on the one-at-a-time approach, through the application of the adjoint-state method, to compute local sensitivity, i.e. the uncertainty on the model output due to small variations of the input parameters, whereas first-order approaches that consider the whole possible variability of the model parameters should be considered.

This theoretical framework and the relevant properties are illustrated by means of a simple numerical example of isothermal ice flow, based on the shallow ice approximation.