



## **Clever particle filters, sequential importance sampling and the optimal proposal**

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Particle filters rely on sequential importance sampling and it is well known that their performance can depend strongly on the choice of proposal distribution from which new ensemble members (particles) are drawn. The use of clever proposals has seen substantial recent interest in the geophysical literature, with schemes such as the implicit particle filter and the equivalent-weights particle filter. Both these schemes employ proposal distributions at time  $t_{k+1}$  that depend on the state at  $t_k$  and the observations at time  $t_{k+1}$ . I show that, beginning with particles drawn randomly from the conditional distribution of the state at  $t_k$  given observations through  $t_k$ , the optimal proposal (the distribution of the state at  $t_{k+1}$  given the state at  $t_k$  and the observations at  $t_{k+1}$ ) minimizes the variance of the importance weights for particles at  $t_k$  overall all possible proposal distributions. This means that bounds on the performance of the optimal proposal, such as those given by Snyder (2011), also bound the performance of the implicit and equivalent-weights particle filters. In particular, in spite of the fact that they may be dramatically more effective than other particle filters in specific instances, those schemes will suffer degeneracy (maximum importance weight approaching unity) unless the ensemble size is exponentially large in a quantity that, in the simplest case that all degrees of freedom in the system are i.i.d., is proportional to the system dimension. I will also discuss the behavior to be expected in more general cases, such as global numerical weather prediction, and how that behavior depends qualitatively on the observing network.

Snyder, C., 2012: Particle filters, the "optimal" proposal and high-dimensional systems. Proceedings, ECMWF Seminar on Data Assimilation for Atmosphere and Ocean., 6-9 September 2011.