[From the American Journal of Science, Vol. XLVI, August, 1893.]

FISHER'S NEW HYPOTHESIS.

By George F. Becker.

[FROM THE AMERICAN JOURNAL OF SCIENCE, VOL. XLVI, AUGUST, 1893.]

ART. XX.—*Fisher's New Hypothesis;* by GEO. F. BECKER.

IN the June number of this Journal* Mr. O. Fisher reaches the conclusion that on an earth of small viscosity, that is a fluid globe, the height of the oceanic tides would be diminished by only a moderate fraction of its height on a rigid earth. He infers that the existence of tides of short period does not indicate a high value for the rigidity of the earth; a conclusion of great interest to geologists, some of whom per-

* Vol. xlv, p. 464, 1893.

haps will not care to undertake an examination of the reasoning employed in reaching it.

Mr. Fisher obtains this unlooked for result by discussion of a formula of Prof. G. H. Darwin giving the height of the oceanic tide relatively to the nucleus on the "canal theory" for a yielding earth, whether the yielding is elastic or not. For comparison Darwin also states the height of the relative tide on the equilibrium theory for the same value of the potential.* The formulas involve the lag of the tide, which disappears when the case of a fluid earth or that of a rigid earth is considered.

Neglecting the lag, the formula for the "canal theory" may be written

$$a_n - r = \mathrm{R}\left(1 - 2\mathrm{A}\,.\,\frac{2g}{5\tau}\right)$$

where r is the radius of the tidal water-surface, a_n the radius of the nucleus, R one-half the total amplitude of the tide on a rigid earth, g the acceleration of gravity, A (which Darwin denotes by E) the greatest semi-amplitude of the bodily tide at the equator and τ is three times the moon's mass into the square of the earth's mean equatorial radius divided by twice the cube of the moon's distance.

The formula for the equilibrium theory under the same conditions is

$$r' - a'_n = \mathrm{R}'\left(1 - 2\mathrm{A}\,.\,\frac{2g}{5\tau}\right)$$

where the primed letters denote quantities corresponding to the same letters unprimed in the other formula.

From a comparison of the full formulas, equally applicable to those given above, Darwin points out that where the one formula gives high water the other gives low water. This is also the main difference between the theories.† Either formula gives the tide on a rigid nucleus when A is zero. For a fluid homogeneous globe A is the same on either theory.

Mr. Fisher draws his own conclusions from an evaluation of $2\mathrm{A}\,.\,2g/5\tau$, which he computes at 2/5 nearly. He infers that on the canal theory the tides cannot be less than 3/5 of their height on a solid globe.‡ He might also by the same

* Phil. Trans., vol. clxx, p. 26, 1879.

† Compare Darwin's article on tides, Enc. Brit., 9th edition, vol. xxiii, p. 354, "tides inverted." The dynamical theory for an earth completely covered by the ocean would give tides of the same height as the equilibrium theory, if the ocean were 3,000 fathoms deep at the equator and shoaled towards the poles. In general its height depends on the distribution of depth. Ibid. section 15.

‡ Proc. Cambridge Phil. Soc., vol. vii, 1892, p. 337.

process have concluded that the tides would show a corresponding amplitude on the equilibrium theory, as appears from the formulas stated above.

Mr. Fisher's computation is incorrect because he takes for A the value which it would have for a fluid globe, homogeneous or not, were there no mutual attraction between the fluid particles. It was for the purpose of dealing with the effect of this mutual attraction that the method of "spherical harmonics" was evolved. The effect in the case of a fluid globe of uniform density throughout on the equilibrium theory, is well known to be an increase in the ellipticity in the ratio 1 to 1–3/5. The ellipticity, e, of the equilibrium lunar tide, in a fluid earth, with this distribution of density, composed of mutually attracting particles is

$$e = \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{a^3 \mathrm{M}}{\mathrm{D}^3 \mathrm{E}},$$

where (as in Thomson and Tait, Nat. Phil.) a is the earth's mean equatorial radius, D the moon's distance, E the earth's mass, and M the moon's mass.*

Since the ellipticity is small, it is easy to see that $2\mathrm{A} = ae$ and therefore also, since for this case $\mathrm{E}/a^2 = g$, substitution for A and τ of the values assigned to them above gives

$$2\mathrm{A} \cdot \frac{2g}{5\tau} = \frac{5}{2}\,\frac{3}{2}\,\frac{a^4 \mathrm{M}}{\mathrm{D}^3 \mathrm{E}} \cdot \frac{2}{5} \cdot \frac{\mathrm{E}}{a^2} \cdot \frac{2}{3}\,\frac{\mathrm{D}^3}{\mathrm{M}a^2};$$

and here the second member reduces to unity by cancellation. In general, therefore, or irrespective of the fluidity of the earth, the quantity $2\mathrm{A} \,.\, 2g/5\tau$ is simply the ratio of the greatest bodily equatorial tide in any special case to the equilibrium tide on a fluid earth. Thus for a fluid earth the canal theory and the equilibrium theory give the same result, viz: no relative tide, or

$$1 - 2\mathrm{A} \,.\, 2g/5\tau = 0.$$

On any theory yet propounded for the tides, the existence of semi-diurnal tides indicates an earth presenting great resistance to deformation. This resistance, so far as the tides are concerned, may be due either to rigidity or to the viscosity of an ultraviscous fluid, some 20,000 times as viscous as hard brittle pitch at 34° F. In the same paper by Darwin quoted above, he comes to the conclusion "that no very considerable portion of the interior of the earth can even distantly approach the fluid state."

Washington, D. C., June, 1893.

* Compare Nat. Phil., section 819.