Implementing positivity constraints in 4-D resistivity time-lapse inversion

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The 4-D inversion technique is a robust method for the inversion of data from time-lapse 3-D resistivity surveys. The smoothness-constrained least-squares equation used is as follows.

$$\left[\mathbf{J}_i^T\mathbf{R}_d\mathbf{J}_i + \left(\lambda_i\mathbf{W}^T\mathbf{R}_m\mathbf{W} + \alpha_i\mathbf{M}^T\mathbf{R}_t\mathbf{M}\right)\right]\Delta\mathbf{r}_i = \\ = \mathbf{J}_i^T\mathbf{R}_d\mathbf{g}_i - \left(\lambda_i\mathbf{W}^T\mathbf{R}_m\mathbf{W} + \alpha_i\mathbf{M}^T\mathbf{R}_t\mathbf{M}\right)\mathbf{r}_{i-1}$$

J is the Jacobian matrix, **λ** and **α** are the spatial and temporal damping factor vectors. **g** is the data misfit vector, **r** is the model parameter vector and **W** is the spatial roughness filter. $\mathbf{R_d}$, $\mathbf{R_m}$ and $\mathbf{R_t}$ are weighting matrices used by the L1-norm inversion method. While the method reduces artefacts by using a temporal roughness filter **M**, it does not constrain the direction of the changes with time.

As an example, we use the data from an infiltration experiment at the Hollin Hill (U.K.) landslide research site where saline solution was sprinkled on the surface to study the effects of fissuring on hydrological processes and landslide initiation. Figure 1a shows the resistivity inverse model from the initial data set. The infiltration started 1.3 hours later lasting until 6.7 hours. Eight time-lapse data sets were collected. Figure 1b shows the percentage resistivity model change between the initial data set and one collected at 7.3 hours. It shows a large region with negative changes in the top two layers, but there is band to the right of the infiltration zone in the top layer with increases of up to 30% which is an artefact. To incorporate a positivity constraint into the inversion algorithm, we first used the model from the standard time-lapse algorithm. For the model cells that show an increase in the resistivity with time, a truncation procedure was used where the resistivities of the different time models were reset to the mean value (corresponding to zero change with time). We then used the method of transformations in the inversion algorithm to ensure that the resistivity of the later time models are always less than the first model. Figure 1c shows the results where the zones with increased resistivity values are eliminated. It also shows the in-filtration pattern in the third layer more clearly.

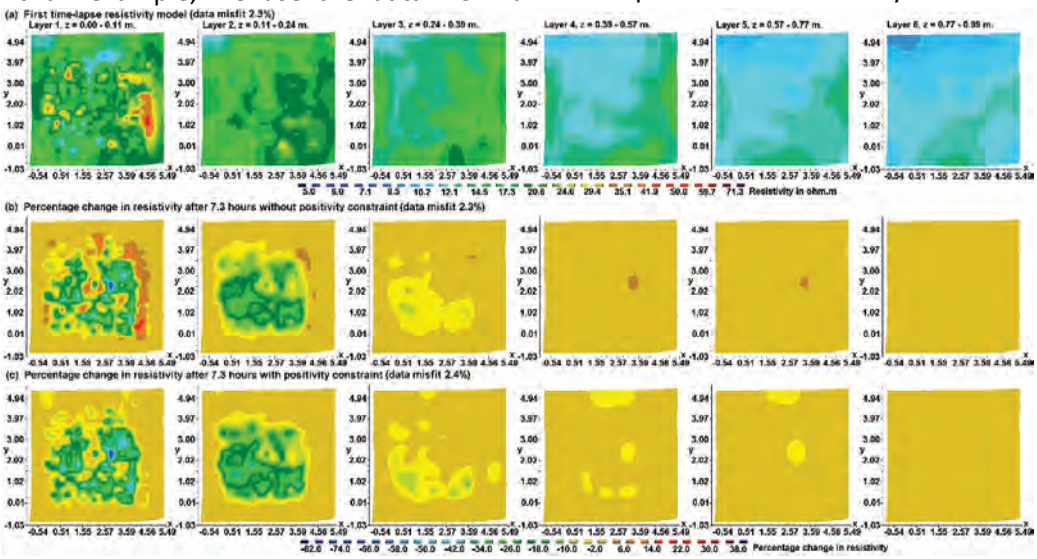

Figure 1: (a) Resistivity model for first data set shown as layers. Percentage change in resistivity from the inverse model after 7.3 hours (b) without and (c) with positivity constraint.